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[illegible]
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[illegible]
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[illegible]
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THE

PHYSICAL REVIEW.

ON THE FREE VIBRATIONS OF A LECHER SYSTEM. IV.[1]

By F. C. Blake and Charles Sheard.

SINCE our last paper we have investigated the effect of the presence of the wooden supports for the capacity plates upon the phase relations and the relative wave-lengths of the various tones. The results of this investigation will be published shortly in The Ohio Journal of Science. We have also investigated the effect of the kind of induction coil and the current-strength upon the wave system. These results are reported here. The influence of the edge-on distance between plates upon the wave system is also here reported.

Apparatus.

Since one of the main objects of our research was to determine the factors that controlled the intensity of the various harmonics and since we had found that the wooden supports introduced into our work phase changes that were appreciable, we found it desirable to eliminate these supports entirely. This was done by supporting the plates by means of Barbour's machine thread No. 18, well waxed to prevent changes of tension due to absorption of moisture. Each of the eight plates was bored with four small holes equally spaced around the circumference, and through each hole two strings were passed and knotted on the back side of each plate. The hole was bored diagonally from the back of the plate to the edge. This left the front face wholly intact. The other ends of the strings were fastened to small windlasses attached to a wooden framework of dimensions $3 \times 1 \times 1$ meters. These windlasses were so distributed on the framework as roughly to cause each of the eight strings from each plate to form an angle of 90 degrees with any adjacent string. Roughly the solid angle round each plate was thus divided by

[1] Read before the American Physical Society, December, 1915.

the string into six right-angled pyramids. In this way it was possible to get the plates accurately lined up and to keep them so. Their alignment was always tested daily. The nearest distance of any part of the system to wood or material other than air was thus 40 cm. except at the oscillator spark-gap and at the receiver. At the spark-gap the hard rubber piece that held the glass tubing necessary for the flow of oil, together with the steel rods that formed the gap, formed an intimate part of the oscillator. At the receiver, the thermocouple T, Fig. 1, was

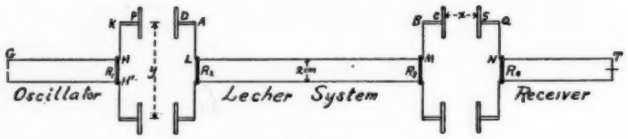


Fig. 1.

enclosed by a hard rubber box 1.9 × 1.6 × 1.3 cm., the walls being about 3 mm. thick. Except at the spark-gap and at the thermocouple the whole vibrating system was surrounded with the one dielectric, air. The small hard rubber pieces, R_1, R_2, R_3, R_4 were assumed to be negligible in their effect. The plates were accurately 5 cm. in diameter and 4.5 mm. thick, to prevent warping under the tension put upon them by the strings. Tension applied by means of other strings attached to $R_1R_2R_3R_4$ gave the necessary tension to the parallel wires. The hard rubber box enclosing the thermocouple T was surrounded with cotton batting to prevent changes of temperature due to air currents. The batting was kept in place throughout the work although it was scarcely necessary except on very windy days. Tension was also applied to T by still other strings leading to a permanent wooden support at a distance of 30 cms. The hard rubber support for the spark-gap was attached to the end of a 2 × 4 inch plank 26 cm. long which was itself attached to an upright 4 × 4 inch plank, 1.1 meter long, on top of which the bottle holding the oil was placed. These wooden supports for oscillator and receiver were movable along a 2 × 6 inch plank by means of iron clamps to which they were attached. In this way the length of oscillator, Lecher system or receiver could be varied at will. As heretofore the distance between the parallel wires was kept constant at 2 cm.

Effect of the Kind of Induction Coil.

In the course of the two or three papers that we have now completed upon this research but not yet published we hope to make clear the experimental conditions that will enable anyone to duplicate our results as a laboratory experiment. In this respect a matter of prime importance is the choice of induction coil.

As previously reported[1] the induction coil that we have constantly used was a 10-inch coil without interrupter or condenser on a 110-volt A.-C. circuit. This gave a fat spark over a length of about 3 cm. at the secondary terminals with a primary current of 10 amperes. Experience had shown that a spark that is almost an arc is the best.

There has lately appeared on the market a new type of induction coil for X-ray work manufactured by the Vulcan Coil Co. of Los Angeles. It is a sort of auto-transformer. It is worthy of special study. For instance, by merely changing the length of the spark-gap the nature of the secondary discharge, *i. e.*, the frequency of alternation in the secondary as well as the current strength in the primary can be changed. Since it operates on a 110-volt A.-C. circuit without an interrupter it was similar to the coil we had been using, so that it could be compared with it. With a given spark-gap it is capable of seven variations in current

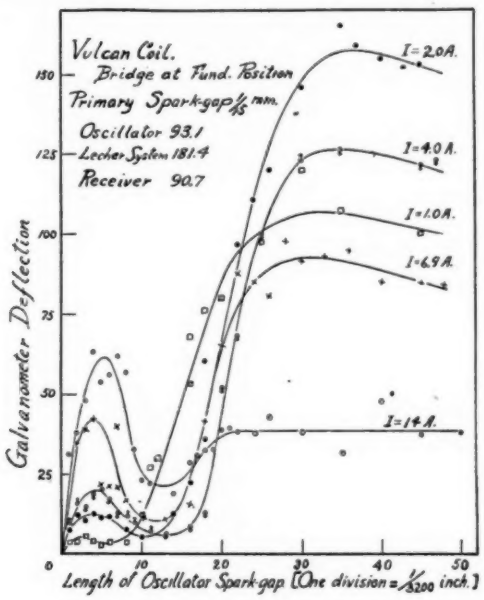


Fig. 2.

strength from 1 to 14 amperes roughly. When it was substituted for the regular coil it gave better results when the water resistance, previously used, was removed. This was accordingly done. A hard rubber micrometer screw-head was attached to the screw that regulated the oscillator oil-gap to enable the length of this gap to be known. The two air gaps were kept constant at 5 mm.

With the Vulcan coil inserted and the water-resistance removed the curves of Fig. 2 were taken. Abscissæ represent the length of the oscil-

[1] PHYSICAL REVIEW, XXXII., p. 534, 1911.

lator spark-gap in oil, the ordinates the intensity of the fundamental tone, represented by the galvanometer deflection for the bridge in the middle of the Lecher wires. Throughout this paper the length of the oscillator will always be expressed as the straight-away length *GH*, the Lecher wires as the length *LM*, the receiver as the length *NT* (see Fig. 1). When the distance "*y*" is known, that is, the edge-on distance between plates center to center, the total length of oscillator, Lecher system or receiver up to the back of the plates can easily be found as needed. The length of the wire *KH* at right angles to the main length is always 1/2 (*y*− 2) cm. At *K* the wire was inserted into a small brass rod, *KP*, 3.5 cm. long and 4.7 mm. diameter, threaded at the end to receive the plate. For all three systems the bends at *H*, *L*, *M* and *N* were rectangular.

For the curves of Fig. 2 the spark-gap of the Vulcan coil was estimated

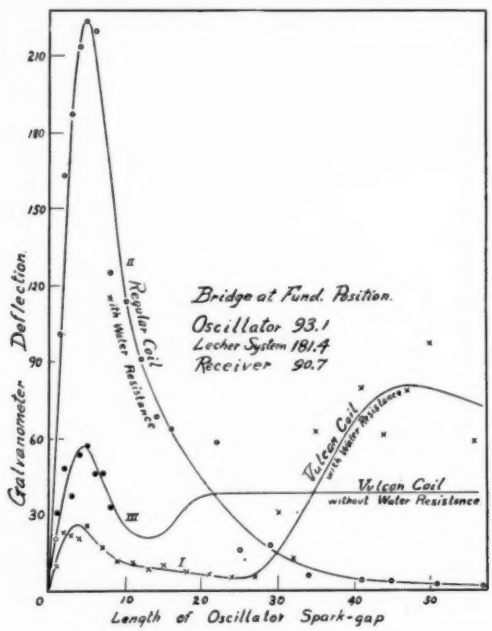


Fig. 3.

at 1/15 mm. In this case the current in the primary coil read 14.0, 6.9, 4.0, 2.0, 1.0 amperes when the coil was adjusted to stops 7, 5, 4, 3, 1 respectively. It is seen at once that there is a rather sharp maximum for oscillator spark-gap varying from 3 to 6 divisions, *i. e.*, from 0.00094 to 0.00188 inch. There is, however, a second broad maximum for much larger lengths of the oscillator spark-gap. The first maximum increases with the current while the second one increases at first and then decreases.

A similar set of curves was obtained when the bridge was placed at the position to give the "third" near the oscillator. Fig. 3 gives a direct comparison between the Vulcan coil and the regular coil. Curve I was obtained with the Vulcan coil, primary current 14 amperes, Curve II. with the regular coil, primary current 13.5 amperes, in both cases the water-resistance being inserted. Curve III. is a Vulcan coil curve without the water-resistance, the portion of the curve for larger oscillator spark-gaps being copied from Fig. 2. In the curves of Figs. 2 and 3, for the oscillator spark-gap in the neighborhood of 50 divisions (1/64 inch) the sparking would occur at *HH'* instead of at *G*, Fig. 1, and the galvanometer would generally read nearly zero. The curves of Fig. 3 at once bring up the question—To what is this second broad maximum due? Since it is not present in Curve II., taken with the regular coil, we are inclined to think it is spurious. It is plain from Fig. 2, moreover, that the ratio

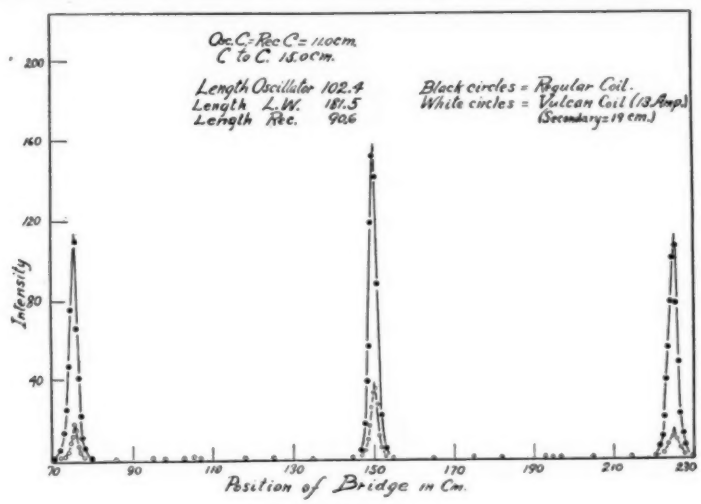


Fig. 4.

of intensities of the fundamental and third is not constant for this second maximum as the current strength is changed, while it is for the first maximum (see also Fig. 5). We found, moreover, that this second maximum is very much more dependent upon the oscillator length and current strength than the first maximum. This is to be expected if it is due to a "slopping-over" of energy from the induction coil upon the vibrating system. We believe that it is brought about in the case of the Vulcan coil because of the relative small inductance of the secondary of the Vulcan coil. This probably enables the secondary to be a part of a vibrating system including the coil and the lead wires to the oscillator and the oscillator itself perhaps. Such a vibrating system would have

a slow period which would tend to dominate effects at the receiver. For the regular coil on the other hand, the inductance of the secondary is, compared with that for the Vulcan coil, exceedingly great. This tends to make the lead wires to the oscillator vibrate as a system independent of the secondary coil and thus enables the free vibrations of the Lecher system (in tune with the oscillator and receiver) to appear to a marked degree as Curve II., Fig. 3, shows.

EFFECT OF PRIMARY CURRENT STRENGTH.

Fig. 4 shows the whole wave-system for the Vulcan and regular coils with the same primary current for a length of oscillator for which only the thirds and the fundamental appear. The advantage of the regular coil is obvious. Fig. 5 was taken with the regular coil for an oscillator length at which the fifths also appear. It is seen that the relation between current-strength in the primary and the tone intensity is strictly a linear one. This accounts for the advantages of a fat spark. We proved the same thing for the Vulcan coil in the case of the third, using the first maximum of course. We have made no attempt to increase the primary current strength beyond 14 amperes for the following reasons. First, the greater the current strength the greater the oscillator deterioration because the greater the amount of carbon soot that tended to deposit on the spark-gap rods. Second, the air-gap rods became red hot much more quickly with increasing current. Third, since we used a Pohl's commutator as a key in the primary circuit the sparking at the mercury cup was rather vigorous for 14 amperes on breaking the circuit.

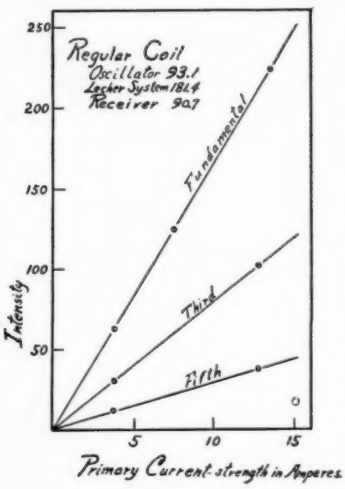


Fig. 5.

EFFECT OF CHANGING THE EDGE-ON DISTANCE BETWEEN PLATES.

Having eliminated the effect of dielectrics in the neighborhood of the capacity plates and having shown the optimum conditions in the choice of an induction coil we reverted to our task[1] of determining the factors that control the intensities of the various harmonics.

At the start, in order to make reasonably sure that the effect of mutual capacity due to the plates edge-on was negligibly small, we chose the

[1] See PHYSICAL REVIEW, Vol. XXXII., p. 534.

distance "y" (Fig. 1) as 35 cm. We had previously[1] shown that the optimum oscillator length for a certain tone varied with the frequency of that tone, being shorter for the higher tones and this was again verified as shown in the curves of Fig. 6. But rather to our surprise the higher

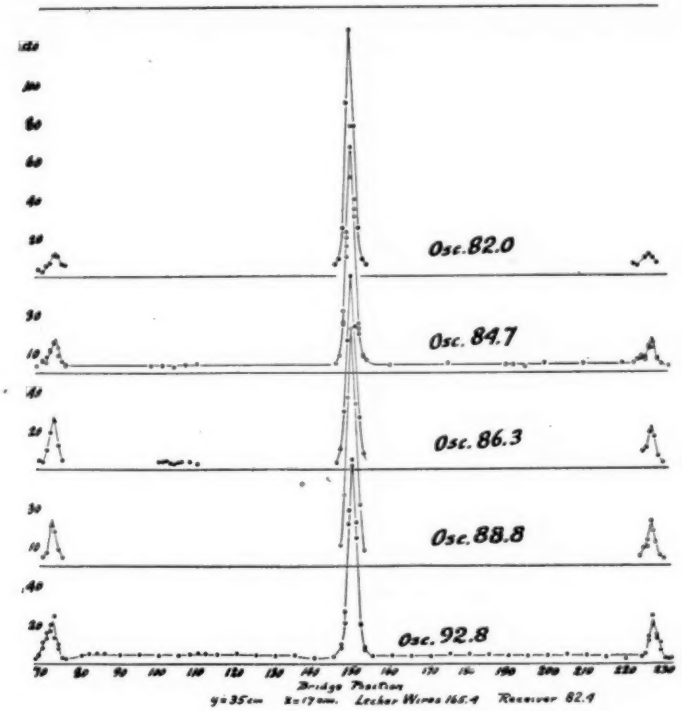


Fig. 6.

tones were very weak. Fig. 6 shows only the thirds and the fundamental, with the maximum value of the thirds only about 15 per cent. of that of the fundamental. Thinking that possibly the very loose coupling (17 cm.) was responsible for the low intensity of the higher harmonics, the coupling was reduced to 10 cm. at each set of plates. This did not help matters. It became increasingly evident that the tone intensity, "I" say, was very much dependent upon the distance "y," the edge-on distance between the plates center to center. Accordingly we had to settle down to a long investigation of the relation between "I" and "y." Since the plates were all suspended by strings this investigation was necessarily tedious. Sets of curves were taken for the following values of y, 35, 20, 15, 12, 8, 7 cm. Some of the results are shown plotted in Figures 6, 7, and 8. For all these curves the receiver was kept very

[1] Physical Review, N. S., Vol. III., p. 217, 1914.

approximately equal to one half the Lecher system. For a given distance "y" the oscillator length was shortened step by step. Then the plates were readjusted to a new distance, new oscillator wires were inserted and again shortened by steps. For any given curve the oscillator spark-gap was adjusted to a maximum for the fundamental; then a curve was taken starting in general with the bridge near the oscillator end of the

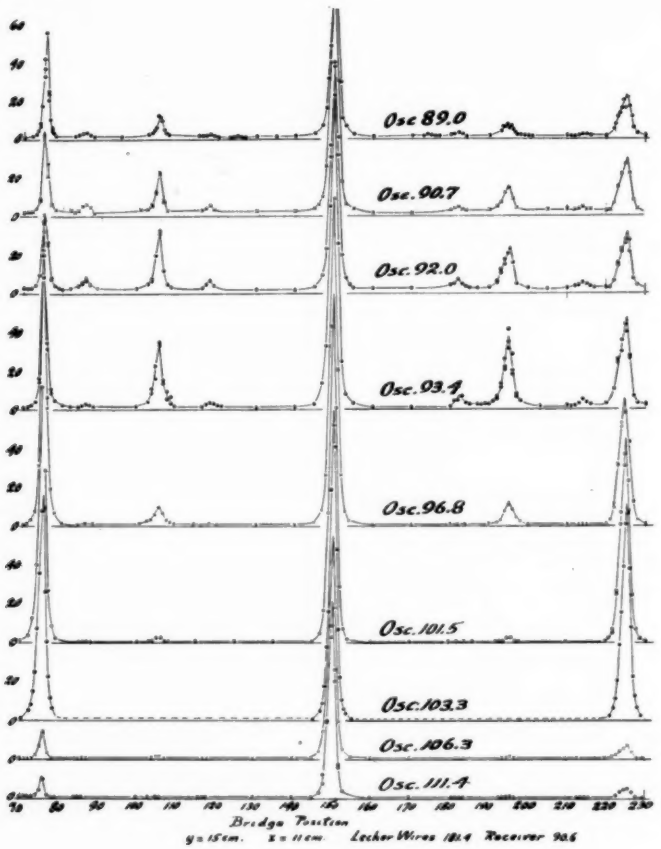


Fig. 7.

Lecher wires. No account is taken in any of these curves of oscillator deterioration. This makes it impossible to compare the various curves directly though one can see clearly that the higher harmonics show only for the lesser distances. Assuming for the moment oscillator deterioration to be zero, many of the curves could be compared directly, for they had the same coupling, and it was plain that for the smallest distances the intensity of the fundamental and the thirds, at least, had decreased considerably. It became plain that there were two factors operating in

opposite directions and that for a certain tone there would be a certain distance "y" at which the tone intensity would be a maximum. Only very roughly though could this distance be determined by the present investigation. A special and still more tedious investigation to determine this has just been completed and will be reported in another paper.

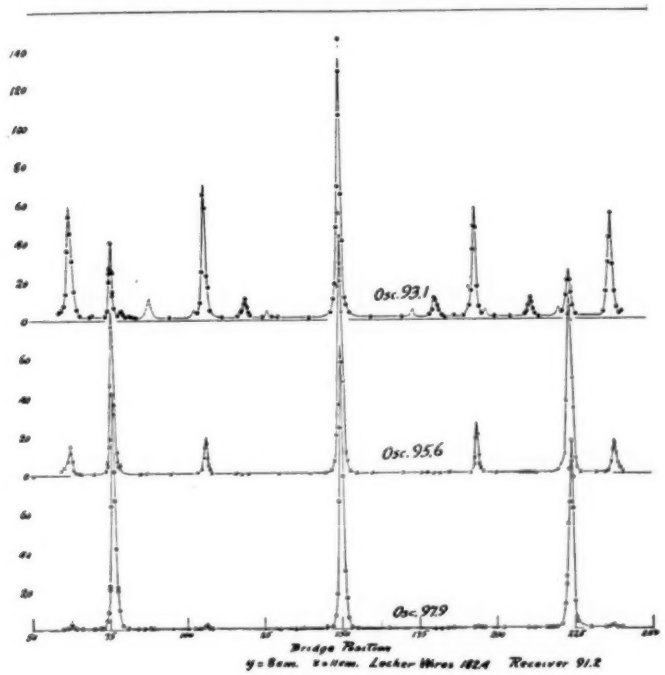


Fig. 8.

Several things of importance fall out of the present study, however, and need to be noted. Of the two factors above mentioned, one is plainly of such a sort that for smaller values of "y" more and more of the electrostatic lines run between any two plates edge-on of the same system for the lower tones, hence less energy gets to the receiver. In a certain rough sense, for comparative results, the ratio y/λ or $(y - 2)/\lambda$ should be a constant for the various tones. This first factor or function we shall speak of as $\theta(y)$. The opposing factor, which we shall call $\phi(y)$, is plainly of such a sort as to introduce a greater phase change for the larger values of "y" in the case of the higher harmonics than for the lower harmonics. This would tend to make the intensity, I, too small for the higher harmonics for a given value of "y."

If we summarize the results of all the curves for a given figure we should get results typified by the curves of Figs. 9 and 10. In Fig. 9,

for $y = 20$, we see that the length of oscillator at which the fundamental and the two thirds are a maximum is practically 100 cm., while for the two fifths it is about 93.5 cm. In Fig. 10, for $y = 12$, the optimum oscil-

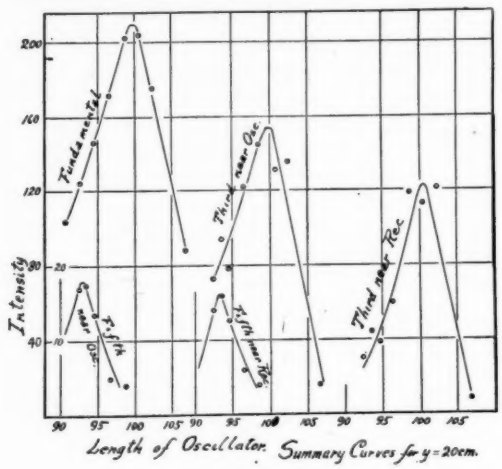


Fig. 9.

lator length for the fundamental and thirds is 96 cm., while for the fifths it is 93.4 cm. and for the sevenths 92.3 cm. There is one essential difference, however, between these two figures in the manner of making the summary. For Fig. 9, the oscillator spark-gap was always adjusted to a maximum

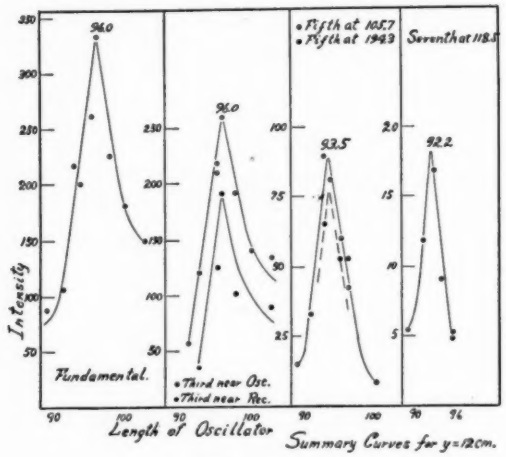


Fig. 10.

for the fundamental and a few readings were taken for the other tones. For Fig. 10, on the other hand, complete spark-gap curves were taken for each tone and the maxima of such curves were plotted. Had spark-gap curves for Fig. 9 been taken the optimum oscillator length for the

fifths would undoubtedly have been over 94 cm. The spark-gap curves for the distance $y = 12$ for the fundamental and the two thirds are shown in Fig. 11. If we summarize the curves of Fig. 11 and other curves not

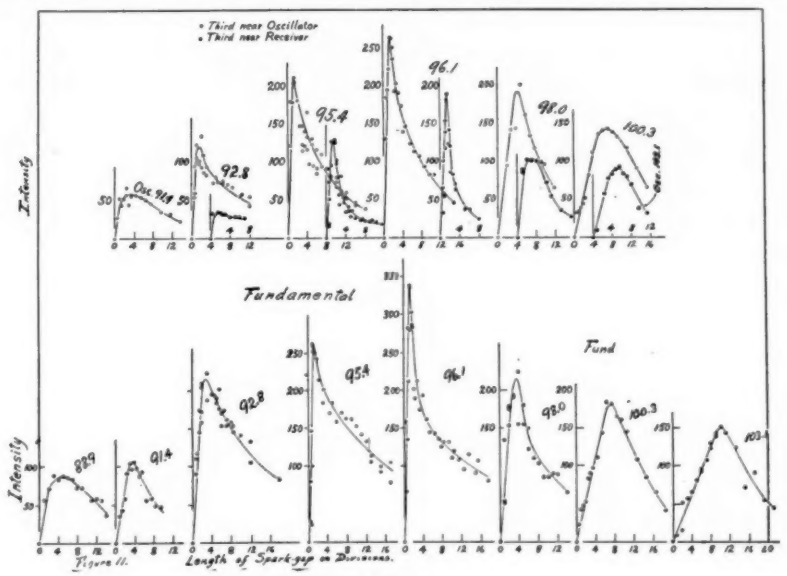


Fig. 11.

shown we get Fig. 12. It proves several points. First, that for each tone maximum intensity occurs at or near the minimum spark length;

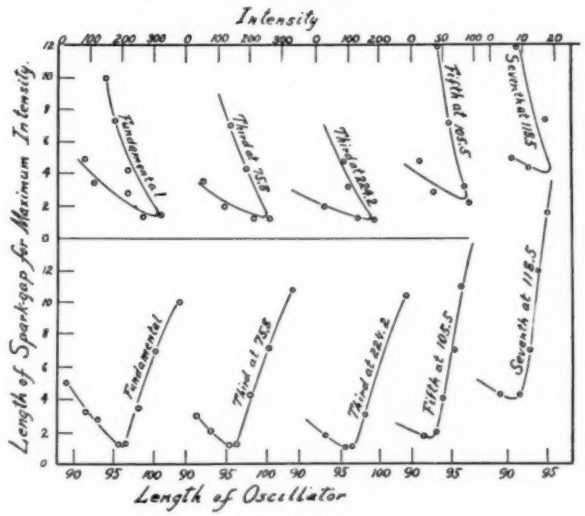


Fig. 12.

second, that this minimum length is practically the same for the fundamental and thirds, about 1.2 divisions, but increases for the higher tones,

being 2.0 divisions for the fifths and 4.0 divisions for the sevenths. Apparently the minimum spark length occurs for oscillator lengths slightly shorter than those that give maximum tone-intensity, at least in the case of the higher tones.

Fig. 13 shows the amounts the peaks for the various tones are displaced as the oscillator length is changed for the case $y = 12$ cm. The amounts the peaks can be varied are very much less than in our previous work owing to the absence of dielectric around the plates. For $y = 12$ the variation is less than for $y = 15$ and for $y = 7$ and 8 it is still less. Moreover, in any case where there is variation the peaks belonging to the same tone vary always in the same direction so that the wave-length is quite independent of the oscillator length. When the plates were mounted on wooden supports this was not the case.[1] Undoubtedly the wave length is equally independent of the receiver length.[2]

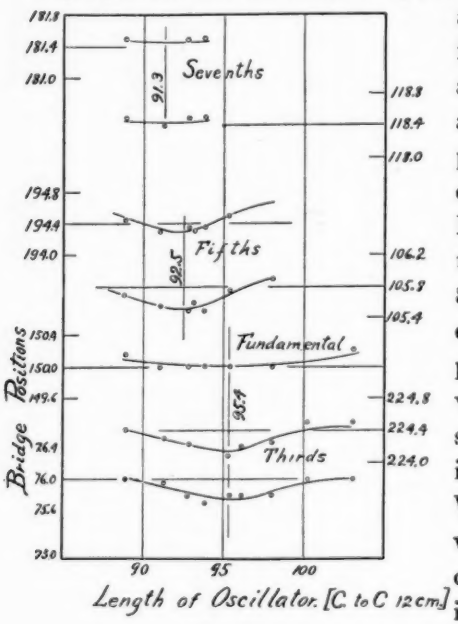


Fig. 13.

The results shown in Figs. 6 to 8 show clearly another advantage of having air the only dielectric surrounding the plates. The anomalous effects portrayed in Fig. 11 of our paper III. on this subject do not now occur. For a given tone the peak nearest the oscillator is the highest, decreasing as the distance from the oscillator increases, due to damping of course. The damping factor can, of course, be determined.

How Does the Capacity of the Plates Vary with the Edge-On Distance Between Them?

Plotting all the peaks of the curves taken for the same coupling, viz., $x = 11$ cm., to a much larger scale than shown here and averaging the internodal spaces the wave-lengths shown in Table I. were obtained. For values of "y" equal to 12 cm. and less each peak was determined by moving the bridge across it by intervals of 1 cm. and returning on the half-centimeter divisions. In general this eliminated oscillator deteriora-

[1] See PHYSICAL REVIEW, N. S., Vol. III., p. 229.

[2] See Figure 5 of our paper III., l. c. The small variation in wave-length due to receiver-length could easily be explained by the presence of dielectrics near the plates.

tion, though when for any reason an observation appeared inconsistent with its neighbors several readings were taken. Such bridge curves would, in general, be equally as good as those shown in our paper I., Fig. 10,[1] and entirely similar to them, though the center of gravity line might not always be quite so vertical as there shown.

The first column of Table I. gives the position, p, of the back side of the Lecher plates near the receiver. The points L and M, Fig. 1, were read directly with a plumb-bob and the 3-meter stick was always adjusted until the midway point between L and M came at 150 cm. Within a fraction of a millimeter the distance $DALMBC$ was always just twice $SQNT$. K, A, B and Q are the points where the copper wire was inserted into the brass rods to which the plates were attached.

In attempting to calculate the value of the end-capacity in terms of the capacity per unit length of the Lecher wires it seemed safest to treat the lengths MB and BC as if they were straight-away prolongations of LM. For it is known[2] that the capacity and inductance per unit length of the wires fulfill the equation

$$L = \frac{\epsilon\mu}{K} = 4\mu \log_e \left(\frac{d + \sqrt{d^2 - b^2}}{b} \right)$$

where "$2d$" is the distance between the axes of the parallel wires and "b" is the wire radius. For air both ϵ and μ equal unity and this equation becomes

$$LK = 1 = 4K \log_e \left(\frac{d + \sqrt{d^2 - b^2}}{b} \right)$$

The equation $LK = 1$ holds for the part LM of the Lecher wires without question. It probably holds for the rods DA and BC also. Hence it seems wise to say that it holds for the portions AL and BM. Although there is a change in diameter at the points A and B (and K and Q) we shall assume that there is no appreciable reflection at these points on that account.

Accordingly the position "p," Table I., can be found by adding $MB + BC$ to the reading of the point M. Of course $MB = 1/2\ (y - 2)$. The lengths, l_s, of Table I. are found from the relation

$$l_s = p - \left\{ 150 + (s - a) \frac{\lambda_s}{2} \right\},$$

where "s" equals the frequency numbers 1, 3, 5, 7, etc., and "a" the corresponding natural numbers 1, 2, 3, 4, etc. Thus "l_s" is the distance between the nearest bridge position and the end-capacity. The last

[1] PHYSICAL REVIEW, XXXI., p. 547.

[2] See, for instance, Abraham, "Théorie der Elektrizität," 2 Aufl., Bd. 1, p. 349.

five columns of Table I. give the ratio K_0/K, calculated for all observed harmonics by means of the formula

$$\frac{K_0}{K} = \frac{\lambda_s/2\pi}{\tan\left(\frac{l_s}{\lambda_s/2\pi}\right)}$$

where K_0 is the value of the end-capacity. Since the frequency changes at most only in the ratio 9 to 1, it is to be expected that the value of K_0/K should be constant, that is, independent of "s." The table shows, indeed, that this is practically the case for small values of "y." The fundamental wave-length λ_1 must, of course, be obtained by extrapolation and this is done by plotting $s\lambda_s$ against s, as shown in Fig. 14. For y

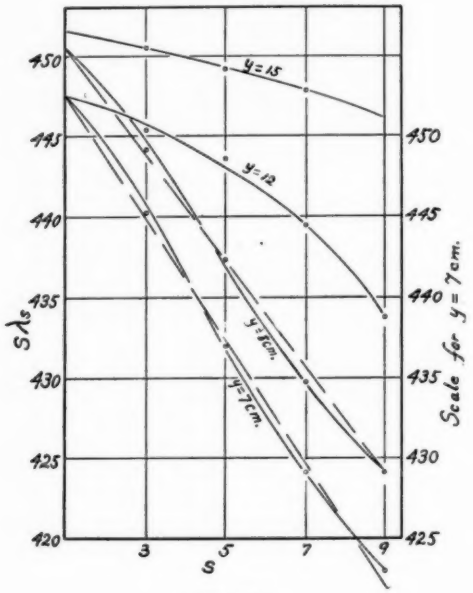


Fig. 14.

equals 7 and 8 cm. the curves shown are theoretical curves obtained for values of K_0/K equalling 16.12 cm. and 15.15 cm. respectively. The points shown are obtained from the observed mean wave-lengths. To emphasize the sinusoidal nature of the theoretical curves straight lines are also plotted. For y equals 12 and 15 cm. the curves are not theoretical curves. Table I. shows clearly that for the larger values of "y" the value of K_0/K is far from constant, apparently increasing rapidly with increasing "s." This goes merely to show that $\phi(y)$ is playing a prominent part in preventing the constancy of K_0/K for these larger values of "y." Naturally the inconstancy of K_0/K will serve to enable one to

judge the form of $\phi(y)$. To show the effect of a slight variation of the wave-length λ, upon the value of K_0/K the comparison between the theoretical and observed values is given in Table I. for the distance $y = 8$

TABLE I.

(x = 11.0 cm.).

		Wave-length.					l_s.					K_0/K.				
p.	y.	λ_1.	λ_3.	λ_5.	λ_7.	λ_9.	l_1.	l_3.	l_5.	l_7.	l_9.	$s=1$.	$s=3$.	$s=5$.	$s=7$.	$s=9$.
250.6	15	451.64	150.19	89.84	63.98	(49.57)	100.60	25.50	10.76	4.63	(1.51)	12.43	13.18	15.27	25.05	(40.70)
249.35	12	446.80	148.44	88.74	62.80	48.2	99.35	25.10	10.65	5.15	2.95	12.65	13.13	15.12	19.65	18.96
247.70	8		148.23	87.37		47.12		23.60	10.33		3.46		15.15	15.15		15.09
		450.50	148.05	87.48	61.40	47.14	97.70	23.67	10.22	5.60	3.42	15.15	15.04	15.42	15.13	15.30
247.25	7	452.50	148.40	87.40	61.30	47.00	97.25	23.10	9.85	5.30	3.25	16.12	15.85	16.24	16.16	16.12

cm. The lower row of figures are the observed values, the upper row the theoretical values taking K_0/K = 15.15 cm. Changing λ_3 0.12 per cent. affects K_0/K 0.73 per cent. Changing λ_5 0.13 per cent. affects K_0/K 1.8 per cent. Changing λ_9 0.04 per cent. affects K_0/K 1.4 per cent. The importance of close measurement of the peak positions can thus be realized.

It will be seen from Fig. 14 that for y equals 7, 8 and 12 cm. the observed points for $s = 3$ all lie below the curves while for $s = 5$ the observed points lie above the curves. One reason for this is that in every case where four fifths were observed the internodal spaces for the fifths either side of the fundamental were slightly less than those for the two outer fifths, *i. e.*, those furthest removed from the fundamental. For $y = 15$ only the two inner fifths could be obtained, hence the point fits on the curve sufficiently well. There must be some second-order effect, possibly the presence of the small ebonite pieces, $R_1 \cdots R_4$, Fig. 1, so near the outer fifths, to increase slightly the outer internodal spaces. Apparently some other second order effect serves to decrease λ_3 slightly below its true value.

Plotting the values of K_0/K against "y" we get the curve of Fig. 15. For $y = 7$ and $y = 8$ the value for $s = 1$ was used. Now the equation

$$\frac{1}{K} = \log_e \frac{d + \sqrt{d^2 - b^2}}{b}$$

holds for the capacity per unit length of the Lecher wires. In our case $2d$ = 2.0 cm. and b = 0.038 cm. Assuming that the plates are simply portions of wires of bigger diameter we can say that this relation holds for the capacity of the plates. If for y equals 7 and 8 cm. we take

K_0/K = 16.12 and 15.15 cm. respectively and extrapolate according to the equation

$$\frac{1}{K_0} = \log_e \frac{d_0 + \sqrt{d_0^2 - b_0^2}}{b_0},$$

where $2d_0 = y$ and $b_0 = 2.5$ cm. we get K_0/K to be 13.69 and 13.24 cm. for y = 12 and 15 cm. respectively. The figures do not differ much from the figures for K_0/K in Table I. for s = 3 which presumably are more accurate than the extrapolated figures for s = 1. The curve of Fig. 15 is the theoretical curve; the points shown are the experimental values of K_0/K for s = 3 taken from Table I.

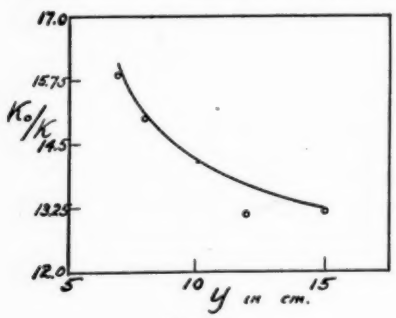


Fig. 15.

A research is now in progress to determine the change in the capacity of a condenser with distance between the plates face on (*i. e.*, changing x).

SUMMARY.

We have shown:

1. That the sharpness of the wave-peaks of a Lecher system depends very much upon the induction coil used as a source of power, the best effects being obtained with a coil giving a fat spark.

2. That the peak intensity is directly proportional to the current strength.

3. That the tone intensity is dependent upon the edge-on distance between the capacity plates, there being for any given tone an optimum edge-on distance.

4. That the logarithmic law for the capacity per unit length of two parallel wires also holds for the change in capacity of a condenser as the distance between it and an equal condenser is varied edge-on.

We are indebted to Mr. Bennett H. Jackson for assistance in taking the observations and to the University for financial assistance.

PHYSICAL LABORATORY,
THE OHIO STATE UNIVERSITY.

THE COMPARISON OF A CERTAIN CASE OF THE ELASTIC CURVE WITH ITS APPROXIMATION.

BY R. W. BURGESS.

IN the usual discussion of the elastic curve in elementary textbooks of physics and engineering, a certain term in the differential equation of the curve is neglected and the curve resulting from this approximation is said to be a good substitute for the real elastic curve. It is my purpose in this note to show by a comparison of one case of this approximate solution with the accurate solution, that the one is not always a satisfactory substitute for the other.

If a straight thin rod in which I is the moment of inertia of a cross-section about a line perpendicular to the plane of bending, and E the constant of elasticity, is bent into a bow by two opposing forces each of magnitude H, acting at the ends of the rod, it is assumed, or deduced from more elementary assumptions, that the resistance to bending at any point is proportional to the curvature of the central axis at that point. The bending moment at any point is therefore proportional to the curvature; taking the central axis when unbent as x-axis, and a perpendicular at its mid point as y-axis, we have $Hy = EI/\rho$, that is, $\rho y = EI/H = a^2$, say, as the equation of the central axis. We can without difficulty integrate this equation, subject to the conditions $y = h$, $dy/dx = 0$ when $x = 0$. Substituting the known value of the radius of curvature in the equation $\rho y = a^2$, we obtain

$$a^2 \frac{d^2y}{dx^2} = - y \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}.$$

As usual, put $dy/dx = p$; then

$$\frac{d^2y}{dx^2} = \frac{dp}{dx} = p \frac{dp}{dy},$$

and therefore

$$\frac{2a^2 p \dfrac{dp}{dy}}{(1 + p^2)^{\frac{3}{2}}} = - 2y.$$

Integrating, remembering that $p = 0$ when $y = h$, we have

$$\frac{2a^2}{(1 + p^2)^{\frac{1}{2}}} = y^2 - h^2 + 2a^2,$$

and hence

$$p^2 = \frac{4a^4}{(y^2 - h^2 + 2a^2)^2} - 1.$$

The length from the top point $y = h$ to the end $y = 0$ is therefore

$$\int_h^0 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_h^0 \frac{\sqrt{1 + p^2}}{p}\, dy$$

$$= \int_h^0 \frac{2a^2}{y^2 - h^2 + 2a^2} \frac{y^2 - h^2 + 2a^2}{\sqrt{4a^4 - (y^2 - h^2 + 2a^2)^2}}\, dy$$

$$= 2a^2 \int_0^h \frac{dy}{\sqrt{2a^2 - (y^2 - h^2 + 2a^2)}\,\sqrt{2a^2 + (y^2 - h^2 + 2a^2)}}.$$

Now put $y = h \cos \phi$, and the length from end to end is seen to be

$$= 4a^2 \int_0^{\pi/2} \frac{h \sin \phi\, d\phi}{h \sin \varphi \sqrt{4a^2 - h^2 \sin^2 \varphi}}$$

$$= 2a \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - \frac{h^2}{4a^2} \sin^2 \varphi}} = 2aK,$$

where K is the complete elliptic integral of the first kind, with $k = h/2a$.

Using the expansion for this integral, as given for instance in Pierce's Shorter Tables, formula 265, or expanding by the binomial theorem and integrating, we get at once

$$l = \pi a \left[1 + \frac{h^2}{16a^2} + \cdots \right].$$

The usual approximation is to omit from the original equation the term $(dy/dx)^2$, which is small if the bow is only slightly bent. The equation is then

$$a^2 \frac{d^2y}{dx^2} = - y,$$

and the solution is $y = h \cos x/a$, which is said to be a good approximation to the solution.[1] If this equation is considered correct, the length of the bow will be

$$l' = 2 \int_0^{(\pi a)/2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = 2 \int_0^{(\pi a)/2} \sqrt{1 + \frac{h^2}{a^2} \sin^2 x/a}\, dx.$$

Put $x/a = \pi/2 - \theta$,

$$l' = 2a \int_0^{\pi/2} \sqrt{\frac{a^2 + h^2}{a^2} - \frac{h^2}{a^2} \sin^2 \theta}\, d\theta$$

[1] See Love's Elasticity, p. 388, or the Encyclo. Math. Weiss., IV., 4, p. 211.

$$= 2\sqrt{a^2+h^2}\int_0^{\pi/2}\sqrt{1-\frac{h^2}{a^2+h^2}\sin^2\theta}\,d\theta$$

$$= 2\sqrt{a^2+h^2}\,E.$$

Using Pierce, formula 266, or expanding by the binomial theorem and integrating, we have

$$l' = \pi\sqrt{a^2+h^2}\left[1-\frac{1}{4}\frac{h^2}{a^2+h^2}\cdots\right]$$

$$= \pi a\left(1+\frac{1}{2}\frac{h^2}{a^2}\right)\left(1-\frac{1}{4}\frac{h^2}{a^2}\cdots\right)$$

$$= \pi a\left(1+\frac{1}{4}\frac{h^2}{a^2}\cdots\right).$$

In the problem of a bent bow, or of a column supporting a weight, the force (*i. e.*, a) and the length l are really the given constants. Our solution should tell us what the deflection is under these conditions. Using the value found for the length of the elastic curve, we find that the deflection h is approximately $4a\sqrt{(l/\pi a)-1}$, and the error of this approximation grows smaller as the deflection decreases. According to the cosine curve, however, the deflection h' is about $2a\sqrt{(l/\pi a)-1}$; that is to say, the deflection deduced from the approximate solution usually considered satisfactory is about one half of the true value and becomes more and more nearly one-half as the deflection grows smaller. I shall show this fact more carefully by obtaining explicit expressions for the two deflections h and h' and working out a few numerical cases.

For the elastic curve, we have

$$l = \pi a\left[1+\left(\frac{1}{2}\right)^2\left(\frac{h}{2a}\right)^2+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\left(\frac{h}{2a}\right)^4\cdots\right]$$

$$\frac{l}{\pi a}-1 = \frac{1}{4}\left(\frac{h}{2a}\right)^2+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\left(\frac{h}{2a}\right)^4\cdots.$$

Now assume

$$\left(\frac{h}{2a}\right)^2 = a_1\left(\frac{l}{\pi a}-1\right)+a_2\left(\frac{l}{\pi a}-1\right)^2+a_3\left(\frac{l}{\pi a}-1\right)^3+\cdots;$$

substitute the value of $(l/\pi a)-1$ from the preceding equation, and equate coefficients of like powers of $[(l/\pi a)-1]$. We thus determine the series

$$\left(\frac{h}{2a}\right)^2 = 4\left(\frac{l}{\pi a}-1\right)-9\left(\frac{l}{\pi a}-1\right)^2+\frac{31}{2}\left(\frac{l}{\pi a}-1\right)^3$$

$$-\frac{185}{8}\left(\frac{l}{\pi a}-1\right)^4+\frac{507}{16}\left(\frac{l}{\pi a}-1\right)^5\cdots.$$

By applying the reasoning of Chrystal's Algebra, Vol. II., pp. 350–353, we may show that this converges at least when $[(l/\pi a) - 1] \leqq \frac{1}{4}$.

Similarly for the cosine curve, we have

$$l' = \pi\sqrt{a^2 + h'^2}\left[1 - \left(\frac{1}{2}\right)^2 \frac{h'^2}{a^2 + h'^2} - \frac{1}{3}\cdot\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\left(\frac{h'^2}{a^2 + h'^2}\right)^2 \cdots\right].$$

Hence

$$\frac{l'}{\pi a} = \left[1 + \frac{1}{2}\frac{h'^2}{a^2} - \frac{1}{8}\frac{h'^4}{a^4} + \frac{1}{16}\frac{h'^6}{a^6} - \cdots\right]$$

$$\times\left[1 - \left(\frac{1}{2}\right)^2\frac{h'^2}{a^2}\left(1 - \frac{h'^2}{a^2} + \frac{h'^4}{a^4}\cdots\right) - \left(\frac{3}{8}\right)^2\frac{h'^4}{3a^4}\left(1 - \frac{2h'^2}{a^2}\cdots\right)\right.$$

$$\left. - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2\frac{h'^6}{5a^6} - \cdots\right]$$

and

$$\frac{l'}{\pi a} - 1 = \frac{1}{4}\frac{h'^2}{a^2} - \frac{3}{64}\frac{h'^4}{a^4} + \frac{5}{256}\frac{h'^6}{a^6}\cdots$$

$$= \left(\frac{h'}{2a}\right)^2 - \frac{3}{4}\left(\frac{h'}{2a}\right)^4 + \frac{5}{4}\left(\frac{h'}{2a}\right)^6\cdots.$$

As before, we assume

$$\left(\frac{h'}{2a}\right)^2 = a_1\left(\frac{l'}{\pi a} - 1\right) + a_2\left(\frac{l'}{\pi a} - 1\right)^2 + a_3\left(\frac{l'}{\pi a} - 1\right)^3\cdots$$

and substitute the value of $(l'/\pi a) - 1$ as given above.

Determining the coefficients as before, we obtain

$$\left(\frac{h'}{2a}\right)^2 = \left(\frac{l'}{\pi a} - 1\right) + \frac{3}{4}\left(\frac{l'}{\pi a} - 1\right)^2 - \frac{1}{8}\left(\frac{l'}{\pi a} - 1\right)^3\cdots.$$

We have, then, two convenient series for h and h', as $(l'/\pi a) - 1$ must be small, that is the force must be not much greater than it is when bending begins to occur. This happens when $l = \pi a = \pi\sqrt{EI/H}$, *i. e.*, $H = \pi^2 EI/l^2$, Euler's column formula. As convenient numerical illustrations, then, we take $l = l' = \pi$, and a successively equal to 1/1.0001, 1/1.001, 1/1.01, 1/1.05 and 1/1.1. It will be noticed that the last value of a gives a rather large deflection, beyond the point where the cosine curve would be considered a good substitute by any one.

We see that for small deflections, the error of the usual solution is nearly 100 per cent. It seems to me, therefore, that the cosine curve $y = h\cos x/a$, with h determined by a formula deduced from this equation, is not satisfactory as an approximate solution, in any case where the length of the bow or column is one of the given physical constants.

l.	a.	h (Deflection Elastic Curve).	h' (Deflection Cosine Curve).	h/h'.	h/l.
3.1416	$\frac{1}{1.0001}$	.03999	.019999	1.9994	.0127
"	$\frac{1}{1.001}$	0.1263	.06321	1.9981	.0402
"	$\frac{1}{1.01}$	0.3915	0.1988	1.969	.125
"	$\frac{1}{1.05}$	0.8065	0.4337	1.86	.257
"	$\frac{1}{1.1}$	1.034	0.596	1.73	.329

The difficulty is of course due to the fact that the deflection is of the same order of magnitude as the slope, which was considered negligible. Errors of this nature, due to dropping terms from a differential equation, probably exist in other physical problems; this one is unusual in that both the exact and the approximate equations can be solved in terms of known functions.

THE EFFECT OF PREVIOUS FILTERING UPON THE ABSORPTION COEFFICIENTS OF HIGH FREQUENCY X-RAYS.

BY S. J. ALLEN AND L. M. ALEXANDER.

A GREAT deal of interest has been created by the work of Rutherford and Barnes on the absorption of the high frequency X-rays from a Coolidge tube at voltages over 100,000. In that work only the so-called "end rays" were used, the great remainder being cut out by previous filtering. It was thought that in this way, the softer components being cut out first, the very small remainder would consist almost wholly of the most penetrating components of the heterogeneous bundle from the tube. They found minimum absorption coefficients for aluminium (.144) and lead (2.06) which did not change for voltages of 120,000 to 175,000. This result was contrary to the existing theory as to the relation between potential and frequency. This method of obtaining the absorption coefficients of the "end rays" is open to question, since in any case the coefficients so obtained only represent a certain "average," and not the hardest components.

Another very important point to remember is, that if the substance used as a filter has excited in it a characteristic secondary radiation, the emergent rays will on the average be softer than otherwise, since there will be superimposed on the primary rays a more or less large proportion

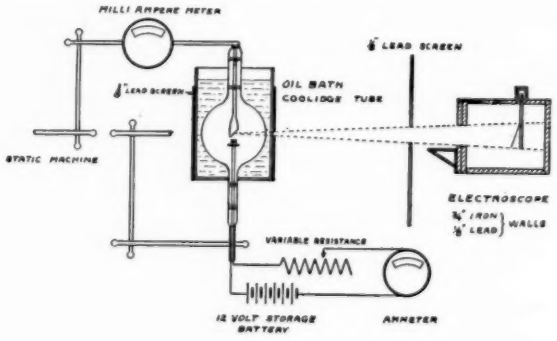


Fig. 1.

of softer secondary rays. If the characteristic radiation is very soft, compared to the primary it will have small effect, but if it is just a little

softer it will have a very large effect. There is every reason to believe that the high atomic weight elements have K radiations whose hardness is comparable to that of the high frequency primary rays used by Barnes and Rutherford, and that their end rays contained in the case of lead filtering a large proportion of lead characteristic K radiation. The present work of the authors' will bring this point out clearly.

The experimental arrangement is simple and is shown sketched in Fig. 1. The Coolidge tube was mounted in an oil bath to prevent danger of puncture. The walls of the glass vessel containing the oil were covered with $\frac{1}{8}$ in. of lead. The primary rays thus had to pass through about 2 in. of oil and $\frac{1}{2}$ in. of glass, before reaching the filter, which would remove partially some of the softer components.

The tube was excited by a large static machine, driven by an electric motor. The conductors were heavy brass tubes to prevent brush discharges.

The current through the heating coil was kept as constant as possible by a variable carbon resistance, controlled by a long insulated handle.

TABLE I.

λ/ρ for Various Filters.

Absorber and Atomic Number.	Filter, 12-inch Spark Gap.							Filter, 9-inch Spark Gap.	
	None.	Al.	Fe.	Sn.	Pt.	Pb.	Bi.	Pb.	Sn.
Carbon.....6	..	..	..	..	..	.12	..	.12	..
Aluminium.....13	.20	.158	.134	.12	.16	.16	.18	.16	.15
Iron.....26	..	..	..	.19	..	.48	..	.45	..
Nickel.....28	.85	.38	.33	.25	.40	.56	.55	.55	..
Copper.....29	..	..	..	..	..	.63	..	..	..
Zinc.....30	1.04	..	..	..	..	.65	..	.65	..
Silver.....47	..	1.38	.92	.87	1.47	1.80	..	2.1	..
Tin.....50	..	1.40	.96	.95	..	2.00	1.93	3.1	2.5
Tungsten.....74	4.50	..	2.42	2.10	1.87	3.60	..	..	..
Platinum.....78	4.70	3.67	3.15	2.40	..	3.50	..	3.9	4.3
Gold.....79	..	..	..	2.30	..	3.50	..	..	..
Lead.....82	3.95	3.07	2.83	2.40	2.64	2.30	2.91	2.2	3.2
Bismuth.....83	4.10	2.90	2.93	2.80	2.78	2.60	..	2.4	3.3

It was found that after about 20 minutes the static machine and conductors reached a constant voltage, and for several hours thereafter the tube gave quite steady readings. During a run the tube was never stopped, but continuous readings of the electroscope taken. A large number of readings were taken and all discordant ones discarded.

The electroscope consisted of an iron cylinder with its axis horizontal and parallel to the path of the rays. The walls were $\frac{3}{4}$ in. thick and the

end nearest to the tube 2 in. thick. A window to receive the rays was covered with a sheet of aluminium $\frac{1}{8}$ in. thick.

The walls were in addition covered with $\frac{1}{8}$ inch of lead. A lead screen placed between the tube and the electroscope, contained a window at which was placed the filter to be studied. The absorptive screens were placed directly in front of the window of the electroscope.

In all cases the filtering screen was made of sufficient thickness to

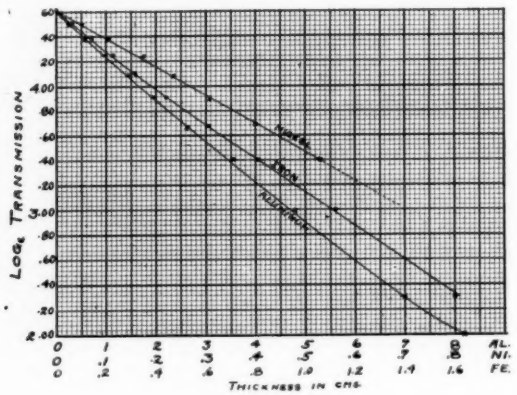


Fig. 2.

bring the initial ionization in the electroscope to the same value I_0. If I_d is the ionization after passing through a thickness d, then the absorption coefficient

$$\lambda = \frac{\log_e I_0 - \log_e I_d}{d}.$$

The mass absorption coefficient $= \lambda/\rho$, where ρ = density.

The results with various filters and absorbers are given in Table I. Two sets are tabulated, one at 12-in. spark gap (about 120,000 volts), and the other at 9-in. spark gap (about 100,000 volts).

In Figs. 2 and 3 are plotted the absorption curves of Al, Fe, Ni, Pt, Au, and Pb, all tin filtered.

In Fig. 4 the values of λ/ρ, for tin, and lead filtered rays are plotted as a function of the atomic numbers.

For the purpose of bringing out the main characteristics of the results in Table I., it will be sufficient to discuss the values for tin, and lead filtered rays. It will be noticed that the values of λ/ρ for the tin filtered are with the exception of Pb and Bi much smaller than the corresponding ones for lead filtered. On the other hand the values of λ/ρ for Pb and Bi, filtered through lead are smaller than those filtered through tin. This is illustrated very clearly by the two curves in Fig. 4, the lead filtered

curve lying well above that of the tin filtered, except for lead and bismuth.

The results for the other filters fall into two classes, those less in atomic weight than tin behaving like tin, and those greater in atomic

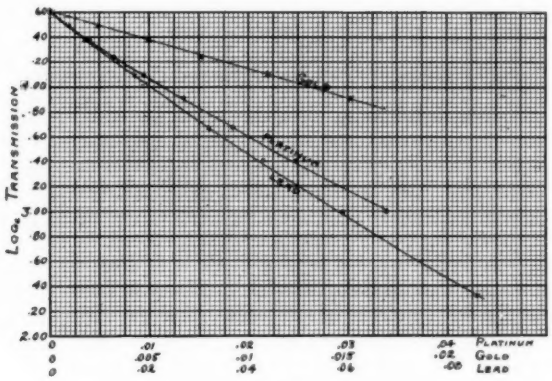


Fig. 3.

weight than platinum behaving like lead. In other words, the rays after being filtered through Pt, Pb, and Bi, are softer than when filtered through Al, Fe, and Sn.

If we compare the results for 9-in. spark gap with those for 12-in.

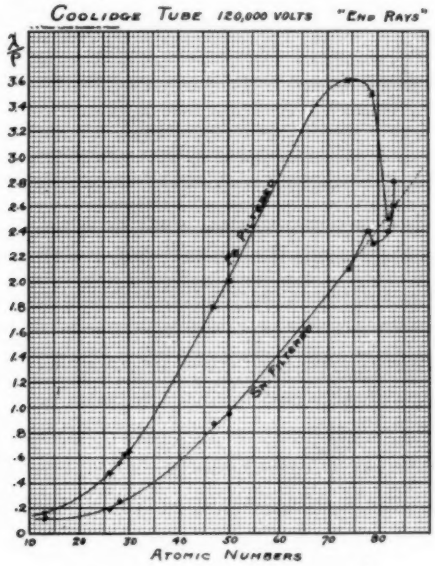


Fig. 4.

spark gap, we see that with a lead filter the rays are apparently not hardened appreciably in increasing the potential, but with the tin filter they show a decided hardening.

The explanation of these peculiarities is clear on the view expressed above. The primary rays in passing through the filters excite in them the characteristic K radiations which are softer than the primary, and therefore the emergent beam will consist in part of a hardened primary and in part of a softened secondary. As the filtering increases the proportion of secondary present will increase in the case of the high atomic weight atoms like lead, but will decrease in the case of lower atomic weight atoms like tin. For low atomic weight atoms like iron or aluminium, any K radiation if present would be too soft to have any effect. As a consequence the results filtered through tin are very much nearer the true values than those filtered through Pt, Pb or Bi.

It is evident from the above results that at a potential of 100,000 to 120,000 volts there is excited in Pt, Pb, and Bi their characteristic K radiations.

The values of λ/ρ for tin filtered indicate the hardest X-rays yet known. The value for aluminium (.12) is considerably smaller than the corresponding one (.144) given by Rutherford and Barnes. We think there is no reason to doubt that if the potential is raised still higher, harder rays can be produced even from a tungsten target.

The absorption curves for tin filtered are practically straight lines, but for lead filtered they show considerable curvature. This shows that the "end rays" for tin are nearly homogeneous, whilst for lead they are not, but contain some softer rays.

The fact that the values of λ/ρ for Pb, when filtered through lead are smaller than when filtered through tin is in accord with the well-known principle that an absorber of the same nature as the radiator always gives a smaller value of λ/ρ when in the neighborhood of the characteristic frequency of that substance.

The secondary rays from the various metals were also observed by direct reflection methods. The metal plate was placed in a horizontal plane and the primary rays projected on it at an angle of 45°. The electroscope was placed so as to receive the secondary rays at an angle also of 45°. The electroscope was carefully shielded by thick screens to prevent any of the primary rays from entering.

The amount of secondary radiation observed was very small in comparison with the primary, and a large proportion of it came from the air in the neighborhood of the electroscope. This amount was subtracted from the total to give the amount due to the radiator. The results are given in the following Table II.

Only a few results are shown for the higher potential as the Coolidge tube broke down at this point, and no measurements could be made on the absorption. Values of λ/ρ are given for the 8-in. spark gap.

TABLE II.

Reflector.	Relative Intensity.		λ/ρ, 8-in. Spark.		
	8-in. Spark.	12-in. Spark.	Al.	Sn.	Pb.
Carbon	114	....	....	....	....
Aluminium	100	100	.25	7.4	5.3
Iron	....	57	....	....	....
Nickel	41	41	.41	3.7	4.4
Copper	36	....	....	....	....
Zinc	30	....	....	....	....
Silver	100	....	....	....	....
Tin	170	118	2.0	11.8	....
Antimony	200	....	....	....	....
Platinum	85	....	....	....	....
Gold	76	....	....	....	....
Lead	62	....	.38	4.0	2.4
Bismuth	45	....	....	....	....

The results for intensity are given relative to aluminium in per cent. These values include the scattered radiation as well as the possible characteristic K radiation. All elements below silver will have only scattered radiation. It is seen that the low atomic weight atoms scatter more than the high, reaching a minimum about zinc. From silver up there is a steady and rapid increase, followed afterwards by a rapid decrease to bismuth. The absorption coefficients for tin show clearly the presence of the K radiation for that element. If the complete values for the 12-in. spark gap could have been obtained they would have undoubtedly shown the maximum displaced further to the high atomic weight atoms. All these results were taken without any previous filtering except through the glass walls and surrounding oil, thus allowing a large proportion of softer rays to act. If the rays had been filtered there would not have been enough secondary rays to measure, with the current through the tube limited to that given by the static machine.

The results shown in this paper can be briefly summarized as follows:

1. The absorption coefficients have been measured for the high frequency X-rays from a Coolidge tube after filtering through various elements, at a voltage of 120,000.

2. These absorption coefficients show clearly the presence of the K radiation of platinum, gold, lead and bismuth.

3. With a tin filter the values of the absorption coefficients for the "end rays" show harder rays than any that have been published up to this time, the value of λ/ρ for aluminium being .12.

4. Experiments on the scattering have been made, showing that the low atomic weight atoms scatter more than the high.

UNIVERSITY OF CINCINNATI, June, 1916.

Addenda.—In an abstract of a paper presented before the American Physical Society, April, 1916, and published in the PHYSICAL REVIEW, Sept., 1916, Hull and Rice give values of the absorption coefficients of the high frequency X-rays from a Coolidge tube reflected from a rock-salt crystal. They clearly show the K characteristic of lead, which begins at a wave-length of 0.149 Å.U. At a wave-length of 0.147 Å.U., the value of λ/ρ for Al is 0.154, for Cu is 0.71 and for Pb is 1.50. At a wave-length of 0.122 Å.U. λ/ρ for Pb is 3.00. These values are in close agreement with those of the authors, which are of course the average over a short range of wave-lengths.

A CALORIMETRIC RESISTANCE THERMOMETER.

By S. Leroy Brown.

THE variation in the electrical resistance of metallic wires with a change in temperature affords an accurate means of measuring temperatures and temperature differences. Nearly all the development of the resistance thermometer has been confined, however, to platinum as the resistance element. Callender and Griffiths' improved methods of measurements and the work at the Bureau of Standards at Washington has made the platinum resistance thermometer a valuable instrument for moderate and high temperatures. The late work of H. C. Dickenson and E. F. Mueller[1] has produced a very sensitive calorimetric resistance thermometer in which the resistance element consists of about 10 cm. of fine (.02-mm.) platinum wire wound in a flat coil. This thermometer, when accurately calibrated, may be used to reproduce the hydrogen scale between 0 and 100° C. to within .002 or .003° C.

In a previous paper,[2] the author has pointed out the possibilities of utilizing the high temperature coefficient of resistance of the metallic oxides to make resistance thermometers. The paper referred to described oxide resistance thermometers made by clamping the solid oxide elements between the connecting wires and gave data which shows that such a thermometer is very sensitive and reliable as long as the contact resistance at junction with connecting wires is unaltered.

The present investigation has to do with an oxide resistance thermometer made by thrusting two wires into a globule of the molten oxide and allowing it to cool so that only a thin layer of the solid oxide separates the wires. Some difficulty may be experienced by the oxide cracking on account of unequal expansion of the oxide and connecting wires. Lead oxide is easily melted in a gas flame and, when fused with iron wires as described above, makes a reliable connection which is permanent when subjected to temperatures ranging from − 10° C. to 120° C. The resistance of a thermometer made as described may range from a few hundred ohms to several thousand ohms at room temperatures. The iron wires are soldered to larger copper wires near the oxide so that the

[1] Bulletin of Bureau of Standards, Vol. 3, No. 4.

[2] Physical Review, February, 1915.

variation of resistance of the leads due to variable immersion is negligible and compensating leads are not necessary. The heat capacity is very small since the globule of lead oxide need not be larger than the head of a pin. The data given in this paper were taken with the thermometer proper enclosed in a tubular glass sheath but a thin metal sheath would allow the resistance to assume more quickly the surrounding temperature. All junctions, oxide to iron and iron to copper, are near together, thereby eliminating the possibility of unequal heating producing thermal electromotive forces.

The electrical resistance of the oxides decreases as the current through them is increased and therefore the oxide resistance thermometer must be excited with a constant current or so nearly constant that the variation in the current does not cause an appreciable change in the electrical resistance. For example, from curve 2 in Fig. 2 a change of .005° C. produces a resistance change of about .01 ohm at room temperature. According to curve (29.3° C.) in Fig. 3, a change of current equal to 2.5 microamperes will produce about .01 ohm change in resistance when the exciting current is one milliampere; or, a change in current of 2.5 microamperes will produce no greater change in resistance than .005° C. change in temperature under these conditions.

The plan of accurately controlling the strength of the exciting current is shown by reference to Fig. 1. The electromotive force across *ab* is

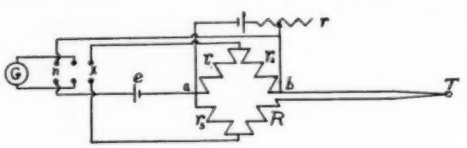


Fig. 1.

maintained equal to the electromotive force of the standard cell *e* by adjusting the variable resistance *r* and this equality is determined by throwing the galvanometer connections to *n* and balancing the electromotive force across *ab* against the electromotive force of the standard cell. The resistance from *a* to *b* is maintained constant; any increase or decrease of the resistance of thermometer *T* is compensated for by a decrease of increase at *R*. The bridge arms r_1, r_2 and r_3 were so chosen that one milliampere of current flowed through the thermometer *T* while the electromotive force across the bridge was maintained equal to the electromotive force of a standard Weston cell.

The data in Table I. are shown graphically by the several curves in Fig. 2, each curve extending over a temperature range of about 30° C., excepting no. 6 for which the temperature range is only from 25 to 28° C.

TABLE I.

Lead Oxide Resistance Thermometer.

Temperature.	Resistance.	Temperature.	Resistance.	Temperature.	Resistance.
−2.45° C.	247.76	25.25° C.	168.28	70.15° C.	101.21
−2.02	245.91	27.90	163.16	74.50	96.48
−1.30	242.91	28.80	161.46	82.30	89.36
− .55	239.66	29.30	160.38	88.00	84.81
0	237.61	30.80	157.42	92.25	81.66
2.40	227.78	32.384	154.26	99.25	76.98
4.30	221.06	35.75	148.33	99.36	76.91
6.72	213.16	45.50	132.18	108.50	71.66
10.00	203.66	51.40	124.04	112.85	69.46
15.55	190.10	58.95	114.48	120.00	66.36
19.67	180.51	66.00	105.91		

The steepness of these curves indicates the sensitiveness of the oxide resistance to temperature variations.

Observations 15–20 were taken by alternately subjecting the thermometer to the temperature of steam and ice. This thermometer has been used by students from time to time during the last year, has been subjected to as much mechanical strain as any other thermometer must

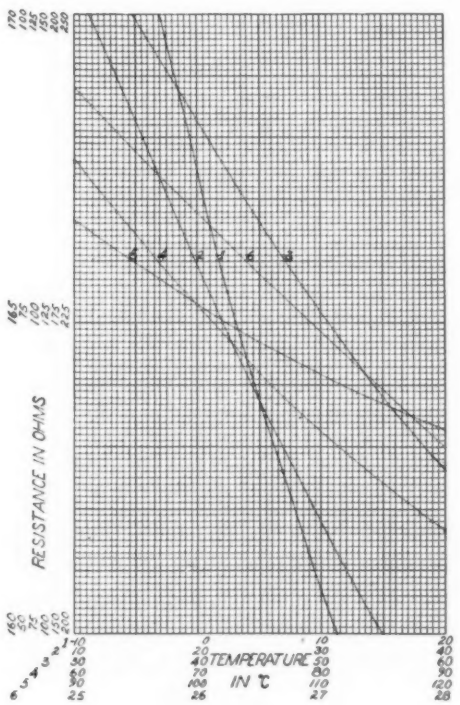


Fig. 2.

TABLE II.

Data Showing Permanency and Reproducibility of Lead Oxide Resistance Thermometer.

No.	Date.	Temp. °C.	Resistance.
1	Dec., 1915.....	99.30° C.	76.95
2	Dec., 1915.....	99.25	77.00
3	Dec., 1915.....	99.45	76.85
4	Dec., 1915.....	0	237.65
5	May, 1916.....	99.35	76.91
6	May, 1916.....	99.33	76.93
7	May, 1916.....	99 40	76.88
8	May, 1916.....	0	237.63
9	Sept., 1916.....	99.20	77.01
10	Sept., 1916.....	99.36	76.91
11	Sept., 1916.....	99.25	76.98
12	Sept., 1916.....	0	237.61
13	Nov., 1916.....	99.68	76.70
14	Nov., 1916.....	99.50	76.82
15	Nov., 1916.....	99.31	76.95
16	Nov., 1916.....	0	237.60
17	Nov., 1916.....	99.31	76.95
18	Nov., 1916.....	0	237.58
19	Nov., 1916.....	99.31	76.95
20	Nov., 1916.....	0	237.62

withstand, and has been subjected to temperatures ranging from — 10° C. to 120° C. Other lead oxide resistance elements have been heated to 200 or 250° C. but a permanent change in their resistance was produced.

Fig. 3 shows the variation in the resistances at various temperatures with a change in the exciting current. The current variations which are permissible at the various temperatures in order that the error produced in the temperature measurement may be negligible must be obtained from the slopes of these curves. A variation of the current, at one milliampere and near zero ° C., not exceeding 1 point in 1,000 would produce an error not to exceed .003° C., and a variation of current at one milliampere and near 100° C., not exceeding 1 point in 1,000 would produce an error not to exceed .0003° C.

The conductivity of the oxide elements made as described above may be decidedly asymmetric. In general the high resistance elements will have a quite different resistance depending on the direction of the current through them. The lead oxide resistance thermometer with which this data was taken showed very little difference in conductivity depending on direction of current. At room temperature, there was a difference of about .1 ohm when one milliampere was reversed, .2 ohm when 2 milliamperes and .3 ohm when 3 milliamperes were reversed. The data were taken for the direction of current which gave the lowest resistance.

A change from very dry to very moist air surrounding the resistance element and a change of pressure of several centimeters produced no measurable change in its resistance. In all probability the greater part

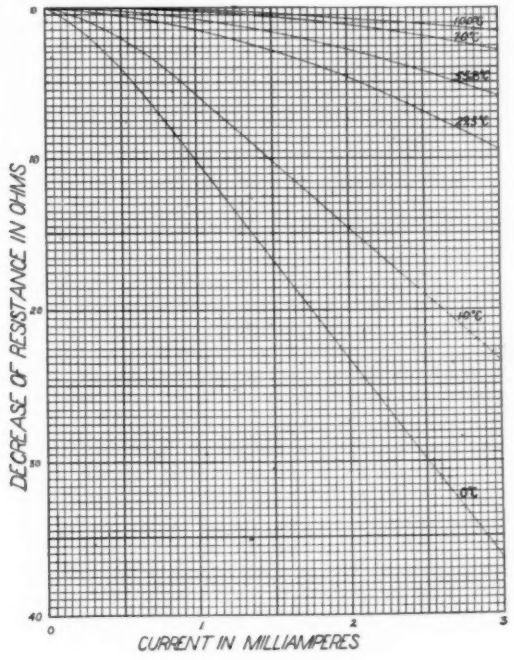


Fig. 3.

of the current was through a thin section of the oxide well inside the protecting shell of hard oxide and, therefore, external changes of moisture and pressure would not be effective.

SUMMARY.

1. A sensitive resistance thermometer for calorimetric work can easily be made which utilizes the high temperature coefficient of resistance of lead oxide.

2. This thermometer does not require compensating leads, has low heat capacity, and since the junctions are close together, unequal heating cannot produce thermal electromotive forces.

3. Temperature measurements can be reproduced with the oxide resistance thermometer to within a few thousandths of a degree between 0° C. and 100° C. The principle disadvantage as compared to the platinum resistance thermometer is that there is no simple relation between resistance and temperature. The calibration must be point by point comparison with a standard and the three fixed points, transition temperature of sodium sulphate, ice point, and steam point.

PHYSICAL LABORATORIES, UNIVERSITY OF TEXAS.

PHOTO-ELECTRIC POTENTIALS FOR EXTREMELY SHORT WAVE-LENGTHS.

BY P. E. SABINE.

THE following is a preliminary report of an investigation carried on in the Jefferson Physical Laboratory to determine the maximum photo-electric potential produced by light in the Schumann region of the spectrum as a function of the frequency.

Professor Millikan's work upon the voltage-frequency relationship in the case of the alkali metals has established the validity of the Einstein formula

$$V = \frac{h}{e}v - V_0$$

as a very close approximation ot the facts for these metals. The results of Kadesh and Hennings[1] using the same method upon other metals give a value of h differing from that of Millikan by an amount considerably greater than the percentage error given by the latter. It is therefore desirable to extend the range of frequencies available for photo-electric measurements upon the other metals.

Moreover, since the magnitude of photo-electric currents increases rapidly with decreasing wave-length, the Schumann region of the spectrum offers an attractive field for photo-electric investigations, while the photo-electric cell ought to prove a valuable aid in Schumann spectroscopy.

The results so far obtained may be summarized as follows:

1. A monochromatic illuminator for use in the region between λ 2300, and λ 1250 has been constructed and calibrated.

2. Voltage frequency measurements have been made upon a single metal, zinc, using light of wave-lengths between λ 1800 and λ 1250.

3. It has been shown that the slope of the voltage-frequency line is quite independent of the surface condition of the emitting metal, provided these conditions remain constant during the measurements.

4. A direct study of the effect of stray light of shorter wave-lengths has shown that it is a source of large error in voltage-frequency measurements.

[1] PHYSICAL REVIEW, Sept., 1916, p. 221.

5. The maximum potentials attained by plates of zinc, cadmium and copper illuminated by light of wave-length λ 1250 have been determined. These measurements, taken with those of other experimenters, for longer wave-lengths give values for the slope of the voltage-frequency line very close to h/e.

APPARATUS.

The monochromator is a modification of Professor Lyman's vacuum grating spectrometer. The essential point of difference lies in the fact that the grating is mounted so that it slides along the focal circle, thus focusing different portions of the spectrum upon the slot through which light shines onto the metal plate in the photo-electric cell. Fig. 1 shows

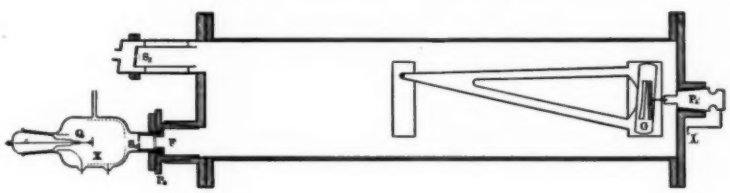


Fig. 1.

the general arrangement. The grating G is mounted upon a movable arm which is pivoted at the center of the focal circle. The slit S_1 and the slot S_2 are adjusted so as to lie upon the same circle. The grating is moved by means of a rack and pinion device. The rack is attached to the movable arm on which the grating is mounted. A small rod through the pinion is engaged by a slot in a sleeve at the end of the taper plug P_1. The grating is shifted from point to point on the focal circle by turning P_1. The position of the index I on a circular scale, attached to the outside of the face plate, determines the position of the grating upon the focal circle and hence the wave length that is incident upon the metal plate. The scale was calibrated by taking a series of photographs of the cadmium and aluminium spark spectra in the region between λ 2300, and λ 1854, and of the spectrum of the hydrogen discharge in the region from λ 1650 to λ 1280.

The photo-electric cell is cemented into the end of the conical plug P_2. The cell is separated from the spectroscope by means of the fluorite window F. The receiving Faraday cylinder is of the usual type made of oxidized copper gauze. The illuminated plate X is a strip of metal 6 × 25 mm., placed well inside the receiving cylinder. It is mounted on the end of a light metal rod, sealed with Khotinski cement into the ground glass stopper which closes the cell. A guard ring Q prevents charges leaking from the receiving cylinder to the illuminated plate.

The latter is connected to a pair of quadrants of the electrometer. Both cell and spectroscope are exhausted by a Trimount Rotary Oil pump. The higher vacuum needed in the photo-electric cells is secured by means of charcoal and liquid air.

SOURCE OF LIGHT.

Considerable time has been spent in attempts to secure a suitable and convenient source of light. What is desired is a source, with a few strong lines well distributed in the region between λ 2300 and λ 1250, that shall be steady and require little attention. Such a combination of qualities has not yet been found in any one source. The metallic vapor arcs of zinc, calcium and cadmium enclosed in quartz have been tried. These spectra possess the requisite lines in this region, but so far it has not been possible to secure working conditions giving the necessary steadiness and reliability. Metallic arcs are difficult to start without heating. Moreover for continuous operation the pressure must be very low. Otherwise the metal soon oxidizes and the arc becomes unsteady. Without a fluorite window between the arc and the spectroscope a very slight leak in the latter soon produces this result. If a fluorite window is inserted, it shortly becomes covered with a film of the metallic vapor even when the arc is 15 cm. from the window and the space between is cooled by a water jacket around the lamp.

On the whole, the hydrogen discharge, while far from ideal, has proven the most satisfactory of the sources tried. It is objectionable in that, with a wide slit, its spectrum becomes practically continuous. Moreover when pure there is little or no radiation of wave-length longer than λ 1650. However, with a trace of air in the spectroscope, carbon bands appear in the region between λ 1650 and λ 2000, so that under these conditions a fairly wide range of frequencies is available. The discharge tube made of quartz was of the usual internal capillary type used by Lyman. It was designed with a view to securing the largest possible current.

The spectrometer case itself served as one electrode. The other electrode was attached to a taper plug of "invar" carefully ground into the quartz tube. The tube was water-jacketed and would carry a current of .2 ampere for 50 seconds without serious heating.

EXPERIMENTAL CONDITIONS.

Large photo-electric currents are desirable in determining maximum potentials. Although with the present form of apparatus only a small fraction of the energy of the source falls on the photo-electric plate, yet the effectiveness of these extremely short wave-lengths is so great that

fairly large currents were obtained. Thus for λ 1600, the most intense region of the hydrogen spectrum, with a slit .03 cm. wide a saturation current giving 10 mm. deflection per second was obtained. For λ 1250, currents one third to one half as great were produced under the same conditions. One millimeter deflection corresponded to .5 millivolt. For the most part electrical conditions were so steady that 1 mm. deflection in 50 seconds could be detected with certainty.

The "back current" due to light reflected from the illuminated plate to the walls of the cell was small. Generally such a current could not be detected at all even under a retarding potential one and a half volts greater than that required to produce zero current. In no case did it amount to more than .5 per cent. of the saturation current. This is probably to be explained by the low reflecting power of the emitting metals for light of short wave-lengths. Hulbert[1] has found that zinc and cadmium have small reflection coefficients in the extreme ultra-violet.

The error due to stray light from the grating, of wave-lengths shorter than that for which the illuminator was set, was found to be large. It will be discussed in a later section.

Surface Conditions and the Voltage-Frequency Relation.

No provision was made in these experiments for preparing surfaces in vacuo. Therefore, it is essential to know how the voltage-frequency relation depends upon the condition of the metal surface. The work of previous investigators seems to show that the only effect of changing surface conditions is a shift of the voltage-frequency line parallel to itself, without affecting its slope. For example Hughes[2] found that exposing a cadmium surface that had been distilled in vacuo, to the action of dry oxygen at a pressure of 14 cm. for 15 minutes, decreased the maximum potentials for all wave-length by the same amount.[3] Millikan[2] also observed that the slope of the voltage-frequency line obtained with a sodium surface originally scraped in vacuo and then exposed to a pressure of .01 mm. agreed very closely with that obtained when the surface was scraped in the highest attainable vacuum and kept under this pressure while measurements were being made. To further test this point experiments were made using a zinc plate with widely different surface conditions. Table I. is typical of the results.

Surface I. had been scraped with a steel tool, and immediately placed in the photo-electric cell. The cell was exhausted in 15 minutes to .001 mm. and the pressure still further reduced by charcoal and liquid

[1] Astrophysical Journal, 42, No. 3, p. 205.

[2] Phil. Trans. A., Vol. 212, pp. 205–226.

[3] Phys. Rev., March, 1916, p. 375.

TABLE I.

Surface.	Max. Volt. λ1280.	Max. Volt. λ1600.	δV.
I.....	5.44	4.32	1.12
II.....	4.64	3.51	1.13
III.....	4.92	3.79	1.13

air. Surface II. had been in the cell for four days at a pressure of .002 mm. and later exposed to the air for 15 minutes. Surface III. had been treated similarly to surface I., but had stood in the cell at low pressure for twenty-four hours. Other experiments in which the variation in the maximum voltage for a given frequency was not so great as those given, showed practically the same difference for the two wave-lengths, so that it seems certain that the change in maximum voltage for a given change in frequency is independent of the surface conditions of the metal, provided these conditions remain constant during the measurements.

SCATTERED LIGHT OF SHORT WAVE-LENGTH.

Millikan[1] has pointed out that the effect of stray light from the optical system giving the monochromatic illumination is in general to decrease the slope of the voltage-frequency curve, by raising the apparent potential due to the lower frequencies. The present attempt to determine the voltage frequency relation in the Schumann region of the spectrum bears out Millikan's suggestion in a somewhat striking manner. The results without color screens briefly stated are these. For wave-lengths λ 1800, λ 1600 and λ 1250 the voltage frequency curve was very accurately a straight line, with a slope however only a little more than one half as great as that called for by the Einstein formula. Table II. gives

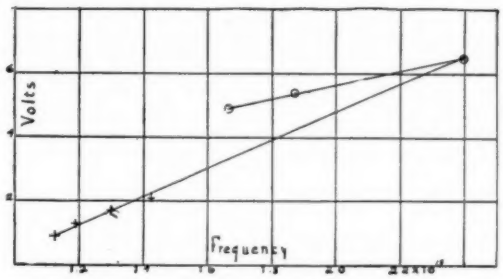
TABLE II.

Wave-length.	Frequency.	Applied Voltage.	Max. Potential.
1800.....	1.66×10^{15}	3.82	4.88
1600.....	1.87×10^{15}	4.32	5.38
1250.....	2.40×10^{15}	5.44	6.50

the values obtained using a scraped surface of zinc. Other determinations with different surfaces gave virtually the same result so far as the slope of the line is concerned. The contact potential difference of 1.06 volts, between the zinc and the receiving cylinder was determined by measuring the positive potential applied to the latter that was necessary to produce the saturation current. These results are plotted (⊙) in

[1] PHYSICAL REV., March, 1916, p. 355.

Fig. 2. The results of Richardson and Compton for a similarly treated zinc surface are shown on the same graph. The slope of the line through the author's points in the Schumann region gives a value for h of 3.6×10^{-27}. That the potential measured for the two longer wave-lengths is larger than that corresponding to the maximum energy of



+, Richardson and Compton's values
⊙, Author's values
} for zinc.

Fig. 2.

electron emission due to these wave-lengths was shown by the following experiment:

With the spectroscope so tight that the leak was less than .02 mm. in twenty-four hours, pure hydrogen was admitted. As has been stated, there are no lines in the Schumann region of the hydrogen spectrum of wave-length longer than about λ 1670. Examined spectroscopically the discharge showed no trace of carbon bands. Yet with the monochromator set for λ 1800, a photo-electric leak almost one tenth as great as that due to λ 1600 was observed. Applying retarding potentials, a voltage current curve was taken. This was found to coincide very closely with the curve due to λ 1600, the nearest and by far the most intense portion of the hydrogen spectrum, and to reach the axis of zero current at very nearly the same value of the retarding potential. In other words, one had the effect due to stray light alone, which in this case was found to be the same as the effect due to the strongest lines in the source of shorter wave-length than that for which the monochromator was set. It appears therefore that without color filters, the maximum potential determined is that at which this stray light effect becomes less than can be detected by the electrometer used and that with a sufficiently sensitive electrometer the voltage frequency line thus determined would be parallel to the frequency axis, at a voltage given by the shortest wave-length that is transmitted by the optical system employed. In the present experiment the trace of air in the spectroscope necessary to produce the radiation at λ 1800 served as a screen to reduce in a

measure the intensity of the stray light of shorter wave-lengths, thus giving a slope greater than zero and less than that due to the lines for which the monochromator was set.

It has not as yet been possible to undertake the task of finding specimens of quartz and colored fluorite to serve as color filters and of accurately determining the limits of their transparency. It has seemed worth while, however, to report this positive result concerning the effect of stray light, in view of the fact that this source of error must exist in all experiments in which color filters are not used.

MAXIMUM POTENTIAL FOR λ 1250.

The foregoing does not apply to the values obtained for the shortest wave-length used since the fluorite window between the cell and the spectroscope cuts out all wave-lengths shorter than λ 1250, the limit of transparency of fluorite. The maximum potential produced when the monochromator is set for this wave-length is therefore a measure of the kinetic energy of the fastest electrons liberated. Since this voltage is large compared with those produced by ordinary ultra-violet light, the percentage error in its determination is correspondingly small. The large range of frequencies between this and the long wave-length limit of most metals is sufficient to afford a good test of the validity of the Einstein formula. Careful determinations of the maximum potential acquired by zinc, cadmium and copper plates were made. In each case the contact potential difference was measured simultaneously by determining the positive potential applied to the receiving electrode necessary to produce saturation of the photo-electric current. The values are given in Table III.

TABLE III.

Metal.	App. Potential.	Contact Potential.	Max. Potential.	Mean.
Zinc.....	5.44	1.25	6.69	
"	5.70	1.17	6.87	6.70
"	5.16	1.35	6.51	
"	5.21	1.60	6.81	
"	5.00	1.63	6.63	
Cadmium.....	5.35	1.25	6.60	
"	5.25	1.28	6.53	6.57
Copper.....	6.05	.63	6.68	6.68
"	6.22	.47	6.69	

The two determinations for copper were made using the same surface, with an interval of about five hours between. Each of the other cases represents a fresh metallic surface. It is to be said that experimental

conditions were much more satisfactory when the measurements upon the cadmium and copper were made than when the earlier work with zinc was done. To make sure that surface conditions had not altered materially between taking the maximum and the contact potentials, the experimental procedure was as follows: First increasing accelerating potentials were applied and the upper part of the voltage current curve was taken to determine the applied voltage that just gave the saturation current. Immediately afterward retarding potentials were applied and the lower end of the current-voltage curve was taken to determine the applied potential just necessary to produce zero current. The saturation voltage was then again determined and the mean of the two values was assumed to be the contact potential difference between the emitting

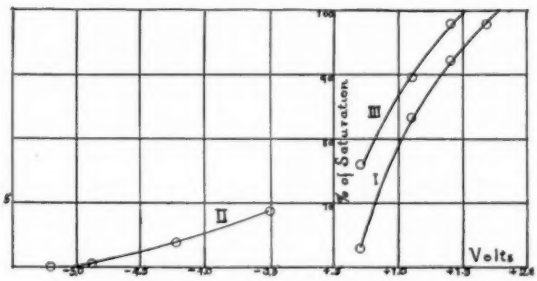


Fig. 3.

and receiving electrodes at the time the maximum potential was determined. The curves in Fig. 3 illustrate the method. They were taken in the order given. The shift of contact potential, .2 volt in the case illustrated was rather larger than usually occurred during a single set of measurements. The time between taking curves I. and III. was about two and a half hours.

The contact potential differences measured in this way with the metal surfaces in vacuo are in good agreement with values given by direct measurement. Thus the mean value for this difference between zinc and the walls of the copper oxide cylinder is 1.4. For copper against copper oxide, the mean C.P.D. is .55 volt. For zinc against copper we have .85 as compared with .84 volt determined directly by Hallwachs for fresh surfaces in air.

In order to use the results here obtained in connection with the longer wave-length determinations of previous observers one must take account of the contact potential difference between the emitting and receiving electrodes in these earlier experiments. Hughes in his work neglected this point entirely so that before using his values they must be corrected for the contact potential difference between the emitting metal and the

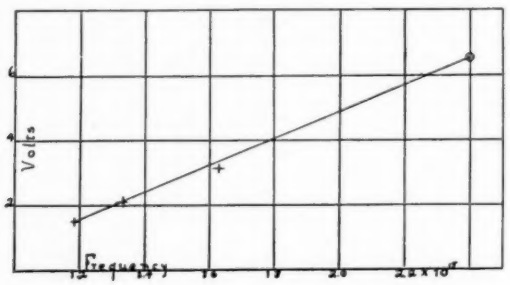
soot-blackened surface of the surrounding walls. One may get an approximate value for this correction as follows: Hallwachs[1] gives as the contact potential difference between zinc and lamp black 1.1 volts. For cadmium and lamp black this value would be .94. Hughes (l. c.) found that exposing his vacuum prepared surface of cadmium to the action of dry oxygen at a pressure of 160 mm. for 15 minutes lowered the maximum voltages for all wave-lengths by the same amount, .25 volt. Hence to compare Hughes's values with those for a surface prepared in air .25 volt must be subtracted. In Table IV. Hughes's values for cadmium corrected by adding .69 volt, *i. e.*, (.94 − .25) are given together with the value here obtained for λ 1250.

TABLE IV.

Wave-length.	Frequency.	Corrected Potential.	
2537.....	1.18×10^{15}	1.52	Hughes
2357.....	$1.33 \times$ "	2.05	"
1849.....	$1.62 \times$ "	3.13	"
1250.....	$2.40 \times$ "	6.57	Author

These are plotted in Fig. 4. The straight line through the extreme points gives a value of h, equal to 6.58×10^{-27}.

In the case of zinc, the mean value of the maximum potential for λ 2537 obtained by Hughes is computed to be .70 volt. Making the



+, Hughes' values. ⊙, Author's value.

Fig. 4.

correction for contact potential and for the effect of air as in the case of cadmium, we have the potential which measures the maximum energy of electrons liberated by this wave-length

$$V = .70 + 1.1 - .25 = 1.55 \text{ volts.}$$

This with the maximum potential for λ 1250 gives

$$h = 6.71 \times 10^{-27}.$$

[1] Handbuch der Radiologie, Band III., p. 349.

The slope of the line in Fig. 2 in which Richardson and Compton's values are used, gives for the value of h, 7.23×10^{-27}.

It is scarcely possible to draw any conclusion from the data for the copper surface, since there are no very satisfactory figures for copper in the longer wave-length region. Richardson and Compton found that copper and bismuth gave unusually low values for the slope of the voltage frequency line. On the other hand, Hughes found that the slope of the bismuth line was very nearly the same as that for cadmium, while Kadesh and Hennings report that for a given frequency all of the metals examined by them including copper acquire practically the same maximum potential when emitting electrons to the same receiving electrode. The experiment with copper was tried only in view of Richardson and Compton's results, to ascertain whether the small slope of the voltage frequency line as determined by them for this metal showed itself in a correspondingly low voltage when the higher frequency was used. The results indicate that there is no reason for supposing that copper differs in any considerable degree from the other metals in the voltage frequency relation of the photo-electric effect.

Taken as a whole the values of the photo-electric potentials for λ 1250, when compared with the results of other investigators, while not affording any very accurate determination of the Planck constant, nevertheless give very good reason to believe that h/e is the slope of the voltage frequency line for zinc and cadmium. It is of course highly desirable that the measurements for both extremities of the available frequency range be made in a single set of experiments with proper precaution to eliminate the known large source of error due to stray light. The necessary changes in the present apparatus are being made for this purpose.

I acknowledge, with great pleasure, my obligation to Prof. Lyman, both for suggesting this research, and for his keen interest in its progress and constant help with the troublesome details of vacuum spectroscopy.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
September 1, 1916.

X-RAY EMISSIVITY AS A FUNCTION OF CATHODE POTENTIAL.

BY DAVID L. WEBSTER.

IN the PHYSICAL REVIEW of June, 1916, I published an account of some experiments on X-rays of definite wave-lengths, characteristic and general, emitted by a rhodium target at definite potentials. One of the results of these experiments was to prove that as the potential was raised the general radiation of any given wave-length would appear first when the potential reached the value which would give a single cathode electron one quantum of that wave-length. This result was similar to that previously observed by Duane and Hunt,[1] and Hull,[2] for the case of tungsten. Furthermore, it appeared that above this potential the intensity of such rays would increase most rapidly at first and then more slowly, the graph soon becoming linear.

Evidently it should be of interest in connection with the problem of the mechanism of X-ray emission to find the emissivity of atoms struck by cathode electrons of a single velocity, as a function of that velocity. These data, however, do not give this emissivity directly, since some of electrons penetrate into the metal, and are slowed down by minor collisions before giving up the quantum of energy that appears necessary to produce any X-rays.

The ideal method of obtaining the true emissivity would be the use of an extremely thin target, which the electrons that did not lose quanta would penetrate with very little loss of speed. As such an experiment presents considerable difficulties in getting strong enough rays for spectrum measurements without melting the target, it seems desirable at present to get the information from known facts as to the loss of speed of electrons in a metal. Fortunately we are enabled to do this by the fact that the chance of any electron's losing a quantum is very small,[3] so that when the potential is increased by a small amount dV almost all the electrons are reduced to the original V without losing a quantum.

[1] Duane & Hunt, PHYS. REV., Aug., 1915.

[2] Hull, PHYS. REV., Jan., 1916.

[3] This point may be deduced from the remarkably low efficiency of an X-ray tube, as shown by Beatty's measurements. See Kaye, "X-Rays" Longmans Green & Co., 1914, p. 106; or Beatty, Proc. Roy. Soc., Nov., 1913.

After this reduction, which occurs in a very thin surface layer, they will behave as though they were striking the target from the potential V. Consequently the increment of intensity by the addition of dV is simply the intensity from this surface layer, and the problem before us is that of finding the chance of emission in one atom in terms of the thickness of this layer and the increment of intensity.

Experimental Laws and Assumptions.—A. As a starting point for this calculation I shall take the law first deduced theoretically by Sir J. J. Thomson[1] and confirmed by the experiments of Whiddington,[2] that the kinetic energy of a cathode particle at a depth x (measured along its path) is given by the formula

$$e^2V^2 - e^2V_x^2 = ax \tag{1}$$

where V is the potential on the tube, V_x the potential that would give the energy it has at the depth x, and a is a constant of the material of the target. This law does not, of course, apply to an electron which has already lost an X-ray quantum, but since such electrons form a very small percentage of the whole, it seems safe to apply it to all as a first approximation.

B. From the results of my experiments quoted above, it appears that the intensity of radiation from rhodium can be represented within the limits of experimental error by the formula

$$I(V, \nu) = k(V - Q) + l(1 - e^{-q(V-Q)}) \tag{2}$$

where $I(V, \nu)d\nu$ is the energy radiated per unit time and per unit of cathode ray current in the frequency interval $d\nu$, V the cathode ray potential, Q the quantum potential $h\nu/e$, and k, l, and q are functions of ν. For example, in the curve for "General radiation .645 to .661 Å." in Fig. 1 of the paper quoted, $kd\nu = 0.0331$ ionization unit per watt, $ld\nu = 0.055$ ionization unit per milliampere, and $q = 0.62$ kilovolt^{-1}, while in the curve for ".571 to .587 Å.," the numbers are 0.0465, 0.065 and 0.77 respectively. The ionization unit used here may be considered as a unit of rate of emission of energy, whose mechanical equivalent varies with frequency but not otherwise. The exact form of the functions k, l and q is not known, since the measured ionizations give comparisons of actual intensity only at a constant frequency and also since the range of frequencies tested was small. Doubtless k increases continuously with ν, and the l term is always comparatively small except near $V = Q$. The spectrum curves for tungsten at constant potentials

[1] "Conduction of Electricity Through Gases," 1906, p. 383.

[2] Camb. Phil. Soc., Proc., Dec., 1911; Proc. Roy. Soc., Apr., 1912.

published by Hull,[1] and Hull and Rice,[2] as well as curves for molybdenum which he has shown me indicate the same general type of law for these materials; though these curves are adapted primarily to the study of I as a function of ν rather than V and do not give any test of the details of equation (2).

Whatever may be said as to the generality of equation (2) as an exact statement, one fact made certain by the work of Duane and Hunt, and

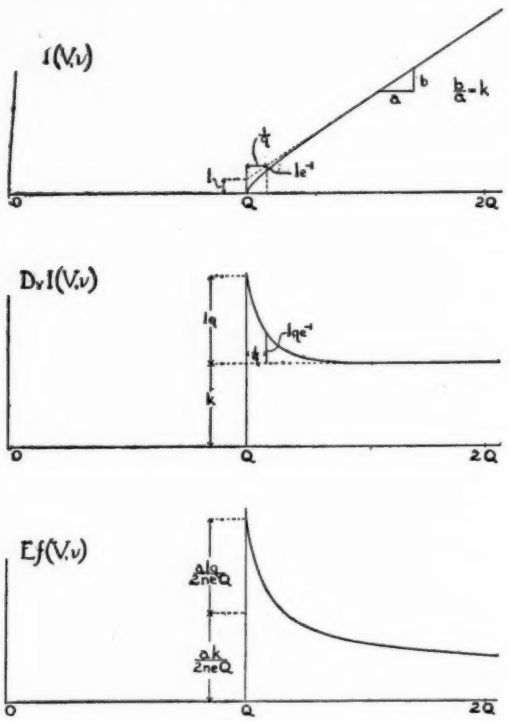


Fig. 1.

Hull, as well as my own, is that the slope of the I, V graph jumps from zero to a finite value at $V = Q$, and another is that in my experiments on rhodium at least the slope is greater at a voltage within one or two per cent. above this potential than at any higher one.[3] These facts, especially the former, are the really essential features of assumption B.

C. Since no X-rays of a given frequency are started by an electron having less than a whole quantum, the energy emitted at each start is

[1] A. W. Hull, Amer. J. of Röntgenology, Dec., 1915.

[2] Hull and Rice, Proc. Nat. Acad., Mar., 1916.

[3] The slight rounding off at the bottoms of these curves is due, as noted in the other paper, to the width of the slit and consequent lack of homogeneity of the rays. It disappears as the slit is narrowed down.

most probably equal to or greater than $h\nu$; and it is evidently equal to or less than the energy of an electron eV_x. These facts, together with the known tendency of radiant energy to become collected in quanta, as shown in the photo-electric effect, lead to the assumption C, that the energy emitted is $h\nu$ itself.

D. It will be assumed that the number of atoms penetrated by an electron in a given distance is independent of V_x and that the probability of starting X-rays of frequency ν to $\nu + d\nu$ in any one of them is $f(V_x, \nu)d\nu$. The problem is that of finding how this function depends on V_x.

Calculation of $f(V, \nu)$.—From the above assumption it is evident that if the potential on the tube is increased from V to $V + dV$, the number of atoms penetrated by an electron in losing energy down to the original V can be expressed as

$$ndx = n\frac{e^2}{a}\{(V + dV)^2 - V^2\} = \frac{2ne^2}{a} VdV \tag{3}$$

where n is the number of atoms per unit distance. Therefore if E is the energy emitted by an atom radiating with frequency ν, if excited at all by this electron, the total energy started by an average electron from these atoms with frequencies between ν and $\nu + d\nu$ is

$$\frac{2ne^2}{a} Ef(V, \nu)VdvdV. \tag{4}$$

Dividing by e one obtains the rate of emission per unit of cathode ray current, from these atoms, which, as we have pointed out above, is the increase of $I(V, \nu)d\nu$ with the increase of potential dV. Therefore

$$Ef(V, \nu) = \frac{a}{2neV} D_V I(V, \nu) = \frac{a}{2ne} \frac{k + lqe^{-q(V-Q)}}{V} \tag{5}$$

The presence of the unknown function E causes some ambiguity in this result, but as we have seen, E is most probably equal to $h\nu$. On the other side of the equation are k, l and q, functions of ν requiring further investigation. For the cases quoted above, k and lq are nearly equal, and q is such that the exponential term disappears at a potential about 20 per cent. above Q. Graphs of I, $D_V I$, and f (assuming $E = h\nu$), for wave-length 0.579 Å. are shown in the accompanying figure.

Conclusions.—From equation (5) it appears that the probability of starting a quantum of X-rays from an atom penetrated by a cathode particle is zero for all energies below the quantum value, at which it rises discontinuously to a maximum. A further increase of energy of the cathode particle decreases this probability. One might expect by analogy with collisions such as those to which Thomson's law (1) relates,

that the chance of losing energy would be less at very high velocities than at low ones, as indicated by (5). Nevertheless the present case is different, in that (5) represents not an average loss of energy to an atom, but a probability of starting a definite frequency of radiation that is emitted only in definite quanta.

In ordinary mechanical systems, and in the systems assumed in dispersion theory, the frequency of oscillation is determined entirely by the constants of the oscillator. It is therefore reasonable to expect that the frequency of the X-ray oscillators must be determined similarly, and that the existence of the short wave limit of the constant potential spectrum means that for some unknown reason no oscillation can occur unless the cathode particle imparts *at least* a quantum of energy to the oscillator. That is, the collision must be within certain limits of obliquity. Obviously these limits will become narrower as the energy of the colliding electron is reduced toward the quantum value. This assumption would therefore make the chance of radiation approach zero, rather than the maximum given by equation (5). Notwithstanding this, the existence of the discontinuous rise in f, followed by a slow decline, is a consequence of the part of the intensity-voltage law of which we are most certain.

An assumption that fits the facts much better is that radiation occurs when the energy imparted to the oscillator is not greater than a quantum, but almost exactly equal to a quantum. In this case, using the equations of Thomson's theory of the slowing of cathode particles, one may readily show that if the force between the oscillator and the cathode electron is an inverse square repulsion only, the chance of transferring energy within a definite small fraction above or below the quantum is inversely proportional to the energy of the cathode electron whenever the latter exceeds the quantum. The same result would follow if the collision were between two hard spheres, though not for some other laws of repulsion such as an inverse cube. While the assumptions and the data are now too uncertain to justify a long mathematical theory on this basis the similarity of this result to equation (5) makes it reasonable to expect that some such assumptions may later be found correct. This means that quantum collisions obey laws that differ from those we are accustomed to even more than one might expect from the mere existence of a short wave limit of the constant potential spectrum.

Summary.—At any rate the above calculations prove the following points:

A. That if radiation occurs in quanta, the probability of its occurrence with any one frequency in any one atom struck by a cathode electron

rises from zero to a finite value when the energy of the cathode electron is increased past the quantum.

B. That for the case of rhodium at least the probability decreases with any further increase of energy of the cathode electron.

C. That these laws of quantum collisions are radically different from those of collisions which merely slow down the electron slightly without producing X-rays.

Jefferson Physical Laboratory,
Cambridge, Mass.

AN ATTEMPT TO DETECT A CHANGE IN THE EMISSIVE PROPERTIES OF PLATINUM AND OF TUNGSTEN AT INCANDESCENCE WITH A CHANGE IN THE METHOD OF HEATING

BY A. G. WORTHING.

IN this work the emissive properties at incandescent temperatures obtained by electric currents have been contrasted with those occurring when the same temperatures are obtained by the absorption of radiation. A possible difference in the general interrelations of the conducting electrons and the atoms for the two methods of heating might be expected to lead to a change in the emissive properties. For both cases it is necessarily true that much of the heat is brought to the radiating layer by thermal conduction. In very thin incandescent ribbons only will the amounts brought thus by conduction differ appreciably. Together with this difference, there will also occur relatively high current densities for the electrical heating method. Though it seemed improbable that one would be able to detect a change resulting from the considerations just stated, it seemed worth while to make the test.

The general procedure consisted in measuring with an optical pyrometer the red and blue brightnesses of very thin incandescent ribbons for the two methods of heating. The terms red and blue brightnesses refer to those values obtained when certain absorption screens are used at the pyrometer eyepiece. The corresponding effective wave-lengths of transmission in the present work were approximately 0.665 μ and 0.470 μ. The blue brightnesses associated with a given red brightness for the two heating methods might be expected to be different or the same depending on whether the effect sought for is real or not. Failure to detect a change by the plan outlined, however, is not conclusive for the red and blue emissive powers in going from one method of heating to the other might both increase or decrease in such a fashion that for the same red brightness, which would then occur at different temperatures, there would occur the same blue brightness.

In the final test on platinum, a bolometer strip between 1 μ and 2 μ thick was mounted just in front of a thick ribbon of the same material.

The thin strip could evidently be heated by either of the two methods. For the electrical method the current densities were of the order of 500 amps. mm^2. The temperatures maintained were in the neighborhood of 1,600° K.

In the final test on tungsten, a thin hollow cylinder made from some sheeting about 0.02 mm. thick was mounted concentrically about a spiral of heavy wire, in an evacuated lamp bulb. For the electrical method the current density used was of the order of 150 amps. mm^2. The temperature maintained was approximately 1850° K.

For both platinum and tungsten the results were negative, the certainty of this being considerably greater in the case of the latter, a consequence of the steadier conditions of operation for this metal. In the case of tungsten the results indicated that, for a given red brightness, the blue brightnesses do not differ with the method of heating by more than 0.5 per cent. In the case of platinum the corresponding value was of the order of 1.5 per cent.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,
NELA PARK, CLEVELAND, OHIO,
November 24, 1916.

SPONTANEOUS GENERATION OF HEAT IN RECENTLY HARDENED STEEL. II.

BY CHARLES F. BRUSH.

IN April, 1915, I had the honor of presenting to The American Philosophical Society a paper on "Spontaneous Generation of Heat in Recently Hardened Steel,"[1] of which the present paper is a continuation. It was shown that a specimen of carbon tool steel, and also a specimen of "high-speed" tungsten-chromium steel after hardening by quenching at a high temperature, spontaneously generated heat in appreciable quantity for at least several weeks, the rate of generation steadily diminishing.

Fig. 1 is a diagram of the apparatus employed. *A*, *B* represent two large silvered Dewar vacuum jars selected to have very nearly equal thermal insulating efficiency. They are supported in a wooden rack inside a thick copper cylinder *C* packed in granulated cork in a wooden box *E*. *D* is a paper extension of *C*, packed with layers of felt by removal of which and the loose copper cover of *C* easy access is had to the Dewar jars. The copper cylinder weighs 52 pounds and its functions are, by reason of its large thermal capacity and high conductivity, to protect the Dewar jars from any rapid change of temperature, and from temperature stratification.

The box *E* is surrounded by a much larger wooden box *F* lagged with felt. By means of a long resistance wire and thermostat, not shown, the air space between the boxes may be maintained at constant temperature.

A' is an airtight cylinder of thin copper, six inches high and two and a half inches in diameter, provided with an open half-inch axial tube also of copper. A small round opening at the top of *A'* permits the introduction of a weighed quantity of water, after which the opening is tightly corked to prevent any change of temperature by evaporation of the water. *B'* is another copper cylinder just like *A'* except that it has a removable top to permit the introduction of the substance whose thermal behavior is to be investigated. The high thermal conductivity of these copper cylinders prevents temperature stratification within them. The

[1] Proc. Am. Phil. Soc., Vol. LIV., No. 217, May–July, 1915.

Dewar jars are filled above the copper cylinders with layers of felt, and granulated cork, and covered with waxed cardboard carefully sealed on to prevent temperature difference inside the jars which would follow unequal loss or gain of moisture by the felt and granulated cork. A small thin glass tube, flanged at top and closed at bottom, is located

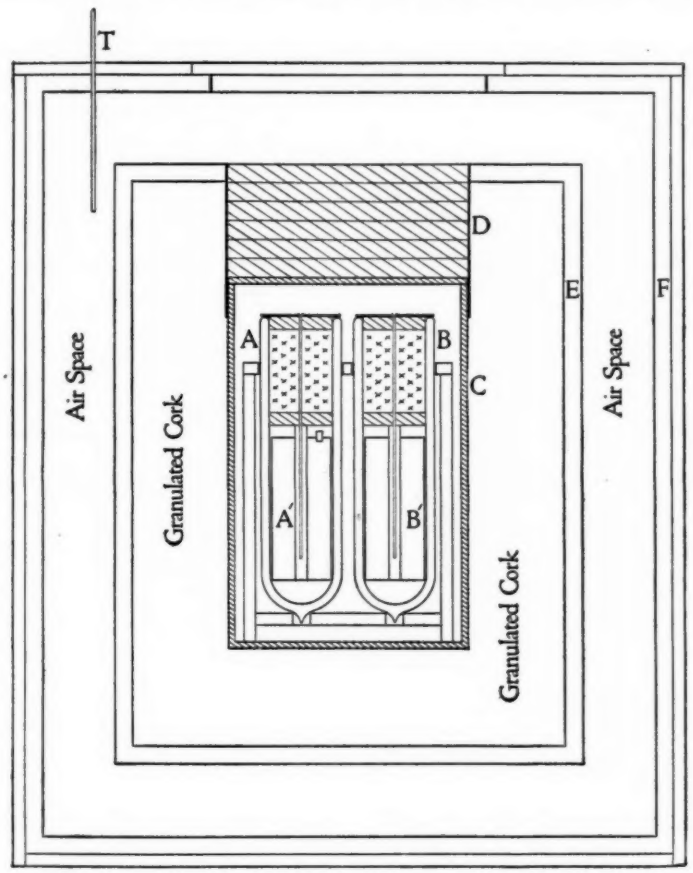


Fig. 1.

in the axis of each Dewar jar and extends from the waxed cover nearly to the bottom of the inclosed copper cylinder. The glass tubes contain the ends of thermo-electric couples of fine constantan, copper and iron wires, one iron-constantan and one copper-constantan junction at the bottom of each tube. The leading-out wires are copper, and connect the thermo-couples with a reflecting galvanometer having the customary reading telescope and scale. Careful calibration has shown that 55 scale divisions of the galvanometer indicate one degree C. temperature-difference between A' and B', and that temperature-difference and

galvanometer deflection are very closely proportional throughout the range used.

In the following experiment A' and B' were removed from the Dewar jars and allowed to attain equal room temperature. Twelve half-inch, round bars of tool steel, five inches long and with machined surfaces were hardened by heating to high "cherry-red" in a reducing atmosphere of a gas furnace and quenching in cold water. The bars then had a thin and strongly adhering coating of black oxide. They were next stirred in a large quantity of water at room temperature, to acquire that tem-

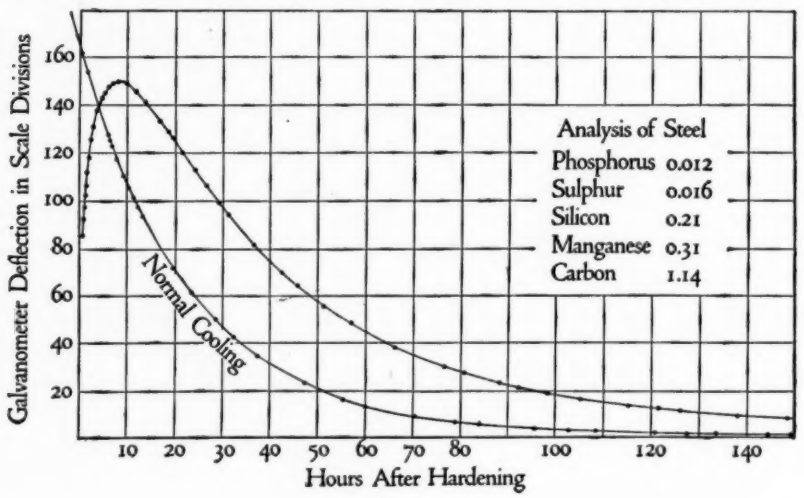


Fig. 2.

perature, wiped dry, and oiled with heavy, neutral mineral oil to prevent generation of heat by further surface oxidation, wiped free of excess of oil and placed in the copper cylinder B'. A weighed quantity of water, also at room temperature, just sufficient to equal the steel bars in thermal capacity had already been placed in A'. The whole apparatus was then assembled as quickly as possible, and galvanometer readings commenced.

The upper curve in Fig. 2 shows the progress of heat generation in the carbon tool steel bars during the first 150 hours after hardening.

The "normal cooling" curve was obtained five or six weeks after the other, and when the generation of heat had very nearly ceased. For this purpose the steel bars were removed, warmed a few degrees, and replaced; then galvanometer readings were made from time to time as before. This curve is plotted in a location convenient for visual comparison with the heating curve, but otherwise might just as well be plotted further to the right. For studying the generation of heat at any point in the upper curve, the curve of normal cooling may be plotted where it will intersect the upper curve at the desired point.

The tungsten-chromium steel, consisting of the same number of bars of the same dimensions as in the first experiment, was treated in the same manner.

Fig. 3 shows the curve of heat generation in the tungsten-chromium steel, and the curve of normal cooling located with respect thereto as in Fig. 2.

The former paper showed that carbon steel when hardened remains

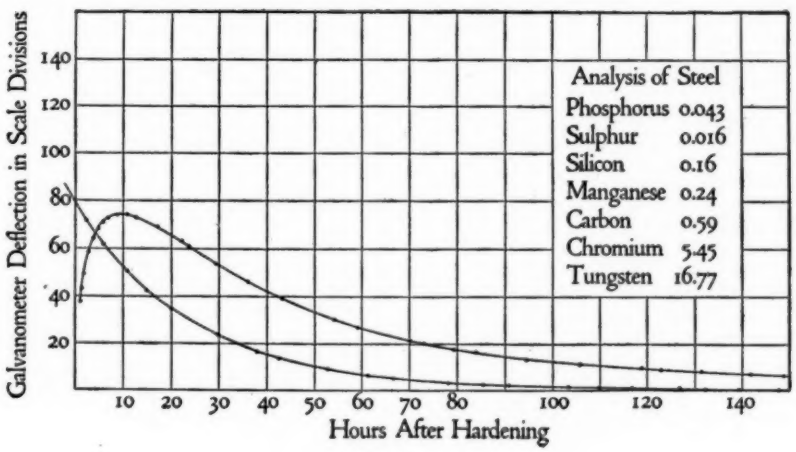


Fig. 3.

very considerably expanded, and that it shrinks progressively when "tempered" to various stages and finally annealed.

A long thin bar of high-carbon steel was found to shrink spontaneously after hardening, to such extent that its progress was easily measured and followed for many days. The spontaneous shrinkage of this steel is plotted in Fig. 4.

It was further shown that while spontaneous generation of heat was probably accompanied by spontaneous shrinking in the specimens of steel examined, yet the shrinking could not possibly be the prime cause of the heat, because the work represented by the heat generated was hundreds of times more than sufficient to effect by compression the reduction in volume that probably occurred. This conclusion, that shrinking was not the cause of the generation of heat, has been questioned; but the more recent experiments, some of which show *no* shrinking during the generation of heat, confirm it.

Six or eight months ago Sir Robert Hadfield wrote an expression of interest in the subject outlined above, and after some correspondence sent two specimens of nickel-chromium steel with the request that they be tested for spontaneous generation of heat after hardening. He also

invited the preparation of a paper for the Faraday Society embodying the results of the tests. Each specimen consisted of twelve half inch round bars five inches long, with machined surfaces, so that results obtained would be quantitatively comparable with those of the other kinds of steel formerly examined.

Specimen *A* was gradually heated in a gas furnace, with reducing atmosphere, while its magnetic susceptibility was observed from time to time by means of a simple induction apparatus devised for the purpose.

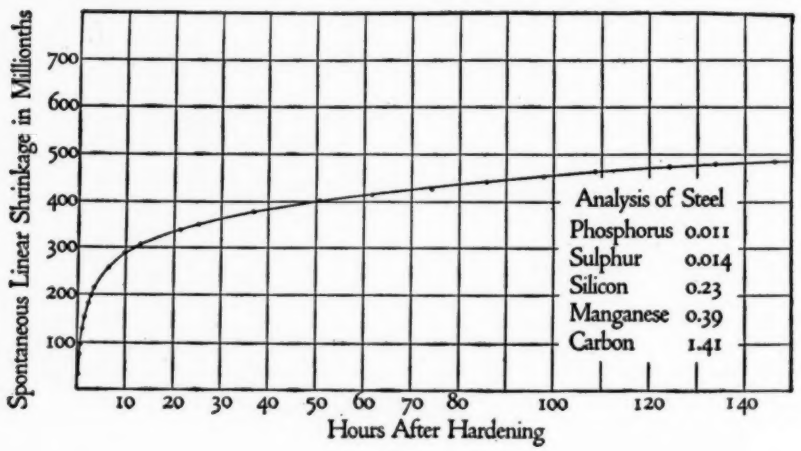


Fig. 4.

Susceptibility remained substantially constant until cherry-red heat was reached, then soon rose ten per cent. or more and again remained constant, at the higher value, for several minutes, then fell off rapidly to nothing. This was taken to indicate complete decalescence, and the proper temperature for hardening. The furnace was immediately opened and the 12 bars quenched separately in water and crushed ice, then stirred in much water at room temperature, wiped dry, oiled and placed in the calorimeter (Fig. 1) all as in the earlier experiments first described.

Fig. 5 shows the progress of heat generation following this first hardening of specimen *A* of the Hadfield nickel-chromium steel. Rapid generation of heat is indicated in the early stages, especially during the first hour, and maximum temperature, the point at which gain of heat was balanced by normal cooling, was reached in about three and a half hours. Obviously the rate of generation had diminished very much by that time and was falling fast, though it still retained considerable value at the end of 150 hours. In general, the curve of "First-Hardening" clearly indicates that the rate of heat generation diminished steadily from the moment of quenching the hot steel.

After a week of cooling, the same bars were again placed in the furnace and heated to a temperature considerably higher than before by continuing the heating fifteen minutes after complete loss of magnetic susceptibility. They were then quenched and treated as formerly.

The other curve of Fig. 5 shows the effect of the higher quenching temperature. Generation of heat was at first much slower than before, the far lower maximum temperature requiring eight hours to reach; but the generation was much better sustained, as indicated also by the

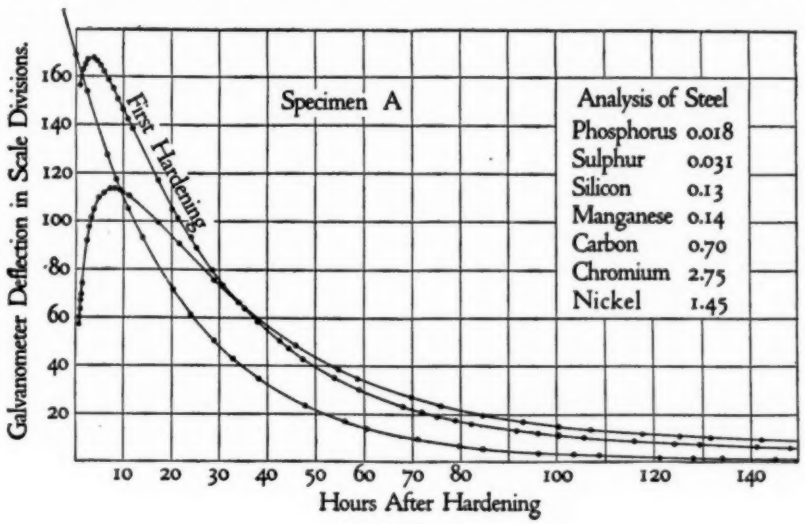


Fig. 5.

crossing of the curves after 35 hours. The total amount of heat generated was not greatly different in the two cases.

Specimen *B* of the Hadfield steel was treated in "First-Hardening" precisely like specimen *A*; but in second hardening was not heated quite so far above the temperature of decalescence.

Fig. 6 shows progress of heat generation in specimen *B* after first and second hardening. The total amount of heat generated was much greater after the second hardening than after the first; *B* differing greatly from *A* in this respect. Probably this was due to the different carbon, chromium and nickel content as shown in the analyses of the two specimens.

The effect on specimen *B* of a moderate increase in quenching temperature was so great that it was thought worth while to quench it a third time, at a temperature somewhat *lower* than in "First-Hardening," and the temperature of complete recalescence was chosen; so that while the quenching temperature of first hardening was just above the critical

temperature of decalescence, that of the third hardening would lie just below the critical temperature of recalescence. In this way light might be thrown on the *cause* of the spontaneous generation of heat.

In pursuance of this idea specimen *B* was again slowly heated in the gas furnace until complete loss of magnetic susceptibility was reached. This was the quenching temperature of first hardening. Then the supply of gas and air were so far reduced that the furnace very gradually cooled. In about twenty minutes the steel commenced to regain its magnetic susceptibility, and a few minutes later had completely regained it. The bars were immediately quenched and treated as formerly.

I had expected to find that the spontaneous generation of heat would

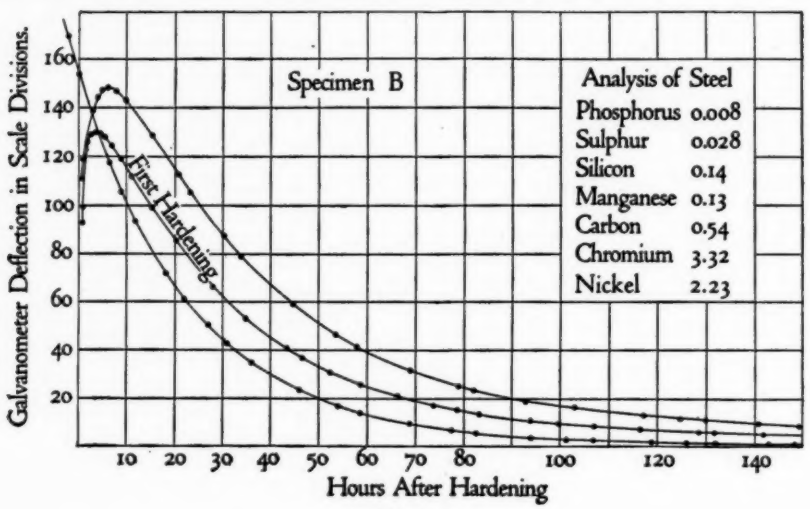


Fig. 6.

be greatly modified, or possibly eliminated by thus quenching the steel below the critical temperature, because true hardening could not have taken place. But it was not so. On the contrary, fully three fourths as much heat was generated as followed the first hardening, and its curve (not plotted) was parallel throughout with that of "First-Hardening."

We may safely conclude then, that the spontaneous generation of heat, at least in the case of specimen *B*, was not associated with the phenomenon of recalescence.

It has been suggested that some, at least, of the spontaneously generated heat found in all my experiments may have been due to chemical action of residual moisture, or oil or both, on the metal or its coating of oxide.

To meet this criticism I again slowly heated specimen *B* until it began to lose its magnetic susceptibility. This was above the temperature

of the previous quenching, and presumably obliterated its effects; it also destroyed the oil, and oily compounds if any, of the previous operation. Then the furnace was closed and, with its contents, allowed to cool four hours. The steel bars were then removed from the furnace, still too hot to handle, allowed to air-cool half an hour longer, stirred in much water at room temperature, wiped dry, oiled and placed in the calorimeter just as formerly. The slow leakage of air into the hot furnace after closing had provided the steel bars with a thin coating of black oxide having the same appearance as that which followed the previous quenchings. The aim of all this procedure was to reproduce as faithfully as possible all the conditions which obtained in previous experiments *except hardening.*

In all former experiments the temperature of the steel had risen considerably during the assembling of the apparatus immediately following the oiling of the bars, as shown by the curve sheets. But in this case the first galvanometer readings were slightly *minus*, showing that the steel was about one fiftieth of a degree colder than the balancing water in the other Dewar jar; and several days passed before approximate equality of temperature was reached.

Hence we may safely say that not the slightest trace of heat was generated in the steel, and that the heat generation observed in former experiments was surely due to hardening.

Before commencing the experiments with specimens *A* and *B* above detailed, a test bar of each lot was prepared by machining its ends slightly convex and polishing, so as to permit of accurate length-measurements by means of a micrometer caliper easily manipulated to constant end-pressure, and easily read to one twenty-thousandth of an inch. The test bars were mechanically held always in the same position between the caliper jaws.

Measurements of the test bars were made after various treatments, and the results are shown in Tables I. and II. The error of measurement does not exceed one unit in the last decimal.

It will be observed that in neither case was there any spontaneous

TABLE I.

Specimen A.

Length of Test Bar.	First Hardening.	Second Hardening.
After hardening	5".0274	5".0354
After spontaneous shrinking	.0274	.0342
After tempering to light straw	.0263	.0293
After tempering to light blue	.0255	.0293
After annealing	.0222	.0250

TABLE II.

Specimen B.

Length of Test Bar.	First Hardening.	Second Hardening.
After hardening	5".0180	5".0151
After spontaneous shrinking	.0180	.0143
After tempering to light straw	.0172	.0093
After tempering to light blue	.0161	.0091
After annealing	.0139	.0063

shrinking after the first hardening, thus demonstrating that the heat generation which then occurred was not due to shrinking.

In both specimens total shrinkage was about twice as great after second hardening as after first hardening, but was divided between the several treatments quite differently. Thus, comparing second hardening shrinkages with first: Considerable occurred spontaneously (none in first); about five times as much followed tempering to light straw, and little or none followed the second tempering, to light blue, though this caused considerable shrinking in both cases of first hardening.

In connection with the first tempering, to light straw color, in both cases of second hardening, an interesting phenomenon was observed, as follows: In all cases of tempering to color, the bar was sandpapered bright, very slowly and uniformly heated until the desired color appeared and then quenched in water at room temperature to arrest progress of tempering. But after this secondary quenching the bars continued to shrink measurably for several hours. The final, stable measurements are given in the tables. Why no further shrinking occurred when the bars were tempered to light blue, is not entirely clear; seemingly, the large shrinkage which followed first tempering, to light straw color, brought about a condition sufficiently stable to withstand further change by the moderately higher temperature required for tempering to light blue.

In conclusion, I cannot at present do better than quote from the closing paragraph of the former paper. "I am led to regard the hardened steel as being in a condition of very great molecular strain somewhat unstable, especially at first. Spontaneous relief of a small portion of the strain causes generation of heat until stability at room temperature is reached. Any considerable rise of temperature, as in tempering, permits further spontaneous relief of strain, or molecular rearrangement, doubtless accompanied by more generation of heat, and so on until annealing is reached."

CLEVELAND, O.,
October, 1916.

THE MEASURABLE QUANTITIES OF PHYSICS.

BY RICHARD C. TOLMAN.

INTRODUCTION.

AS any branch of science develops, its considerations become more and more deductive in their nature, the ideal form of scientific exposition apparently being one in which all the important relations are logically derived from a small number of independent general principles or postulates and all the terms used are defined with the help of a small number of indefinables.[1] The time is already ripe for a much more comprehensive and systematic treatment of the field of mathematical physics than has hitherto been attempted, and the completion of this task would make it possible to derive all the equations of mathematical physics from a few consistent and independent postulates, and to define all the quantities occurring in these equations in terms of a small number of indefinables. The purpose of this article is to discuss from a somewhat general point of view the nature of the quantities which occur in the equations of mathematical physics and to consider a set of indefinables for their definition. We shall thus hope to help in the preparation for that more complete systematization of mathematical physics which is undoubtedly coming.

We shall find it possible in agreement with the work of others to distinguish two general classes of quantity, those having *extensive* and those having *intensive* magnitude, and shall consider the methods necessary for measuring these two quite different kinds of magnitude. We shall then discuss the method of choosing a set of fundamental kinds of quantity, which will serve as indefinables for defining all other kinds of quantity, and shall find that the *number* of independent kinds of quantity which must be taken as fundamental is apparently independent of the particular set that is chosen. Contrary to previous practice, however, we shall find it desirable to choose as fundamental, only quantities which have *extensive* magnitude, since we shall then be able to introduce a

[1] The term "indefinables" is used in its technical sense, and by it we shall understand certain entities which are not defined in the particular discussion undertaken, but for the purposes at hand are assumed rather to be matters upon which there is general agreement. This does not of course in any way prejudice the possibility or desirability of their definition in some more fundamental treatment.

number of important simplifications. Besides this set of fundamental kinds of quantity, we shall also find it necessary to have a set of indefinable operations for use in the definition of quantities, and we shall briefly consider these further indefinables and the definitions which can be obtained with their help. Making use of our definitions of the different kinds of quantities we shall then briefly consider the choice of units for their measurement. In conclusion we shall discuss the impracticability of trying to reduce the number of fundamental kinds of quantity and shall also consider what kind of developments in science will necessitate a further increase in their number.

§ 1. On Magnitude and Quantity.

In accordance with the so-called absolute theory of magnitude as presented by Russell[1] we shall consider *magnitudes* as indefinable *terms* which are capable of entering into the *relations* of "*greater than*" and "*less than.*" The relations "greater than" and "less than" are *assymetrical* and *transitive* and are taken as indefinable. Every magnitude bears a peculiar relation to some particular concept such that we say it is the magnitude of that concept; and magnitudes are said to be of the same kind when they bear this relation to the same concept. Only magnitudes of the same kind can enter into the relations of greater and less.

Using the language of this theory we may speak of magnitudes of pleasure, magnitudes of temperature, magnitudes of volume, etc., each of the concepts pleasure, temperature, volume, etc., having a set of magnitudes of the same kind associated with it. And we can say that one magnitude of volume is greater or less than some other magnitude of volume, but cannot relate it of course in this way with a magnitude of temperature or of pleasure.

Quantities are to be regarded as magnitudes which have been particularized by the specification of spatial or temporal conditions. Thus the statement of the temperature in a particular beaker at a particular time, is a quantity, this quantity having a definite magnitude as one of its attributes. Thus we see that magnitude is a more abstract conception than quantity. We shall say that one quantity is greater than another when its magnitude is greater. We shall say that two quantities are equal, however, when they have the *same* magnitude. To speak of equal magnitudes would be meaningless.

[1] Bertrand Russell, "The Principles of Mathematics." Cambridge, 1903.

§ 2. EXTENSIVE AND INTENSIVE MAGNITUDE.

If we regard the different kinds of quantities that are used by physicists, we find that they fall with reasonable lack of ambiguity into two general classes, those having a certain additive nature so that a given quantity can be regarded as being the sum of a number of smaller quantities of the same kind, and those which have no such additive properties. We say that quantities of the first class have *extensive* magnitude, and that quantities not having an additive nature have *intensive* magnitude.

The distinction between these two kinds of quantity can be made clearer by examples. Volume is typical of quantities having extensive magnitude. A given volume may be regarded as composed of smaller volumes into which it can be mentally divided, and similarly quantities of length, mass, energy, entropy, etc., will be seen on consideration to have extensive magnitude. Temperature, density, magnetic permeability, dielectric inductivity, etc., are quantities which we cannot successfully regard as being the sum of smaller quantities of the same kind and such quantities are said to have intensive magnitude.

In order to decide whether a given quantity has extensive or intensive magnitude it is sometimes helpful to see if the simultaneous presence of two systems, each having a definite quantity of the kind in question, can be regarded as giving a larger system with twice the quantity; if so that kind of quantity has extensive magnitude. For example, consider a system composed of two pieces of platinum, each having a mass of 10 grams; this gives us a larger system of 20 grams, and we say that mass has extensive magnitude. On the other hand, if each piece of platinum had a temperature of 100° absolute, we should not say that the temperature of the total system was 200°; indeed the larger system also would have the same temperature 100° and we say that temperature has intensive rather than extensive magnitude. As another example, two pieces of platinum, each with a density of 21 grams per cubic centimeter, does not give us a larger piece of platinum with a density of 42 grams per cubic centimeter, and density also is a quantity having intensive magnitude.

Whether this classification of quantities into those having extensive and intensive magnitude is perfectly unambiguous is perhaps difficult to decide. The classification is usually simple, however, and like many other classifications of science this is one which would still have great value even if there should turn out to be borderline quantities of such a nature that it was hard to apply the criterion.

§ 3. THE MEASUREMENT OF PHYSICAL QUANTITIES.

The method of measuring quantities which have extensive magnitude is more or less obvious. Since these quantities have an additive nature, we may pick out more or less arbitrarily some particular quantity as having unit magnitude and then find experimentally how many of these unit quantities must be taken together in order to obtain a quantity of the same magnitude as the one in question. Thus we can measure the length of a building by finding how many foot rules must be laid end to end in order to get a length of the same magnitude.

In the case of quantities with extensive magnitude, however, we may distinguish for purposes of measurement two quite different groups. In the first class we may put those quantities which can be regarded as composed of a finite number of distinct and identical parts. For the measurement of these quantities the most natural unit quantity to choose will be one of these individual parts, and the measurement of the quantity will be a process of *counting* the number of these discrete units necessary for the composition of the whole system. For example, the quantity of apples in a barrel may be measured by counting the number present. Quantities of this kind are often used in the natural sciences, as for example in the correlation of the pressure of a gas with the *number* of molecules it contains or in statements as to the *number* of petals in a given species of flower. It should be noticed that quantities of this simple kind which are measured by counting are associated with a discrete series of magnitudes which can be put into one-to-one correspondence with the ordinal numbers.

Quantities having extensive magnitude and belonging to the second group cannot be regarded as the sum of a finite number of parts, and in this case the magnitudes of a given kind form a continuous series and can only be put into one-to-one relation with the whole series of real numbers. Volume is an example of such quantities, it not being in accordance with our present ideas of space to consider a volume as composed of a finite number of parts which could not be further divided. For the measurement of extensive magnitudes of this kind, the choice of unit will usually be somewhat arbitrary, but the additive nature of all quantities with extensive magnitude will permit the construction from unit quantities of a whole having the same magnitude as the quantity to be measured, and the number of unit quantities used will be the desired measurement. In making such measurements, in general of course it will not turn out that our unit quantity is of just the right magnitude to fit, so to speak, an integral number of times into the quantity to be measured, but it will not be necessary to discuss here the

well known methods of approximation which must then be employed.[1]

As for quantities with intensive magnitude, since they do not have an additive nature, their measurement must be brought about by some device in which the magnitudes to be measured are put into a one-to-one correspondence with a series of quantities having extensive magnitude. For example, we might try to measure quantities of pleasure by correlating pleasurable experiences with the money spent in obtaining them. In fact such a correlation is tacitly but pretty generally adopted in a commercial age, and in spite of obvious defects it certainly has the advantage of definiteness. In the case of temperature, a typical physical quantity with intensive magnitude, various methods of measurement have been devised, such for example as the correlation of magnitudes of temperature, with the varying lengths of a thread of mercury, or the varying volumes occupied by a perfect gas or other quantities having extensive magnitude. In our discussion of fundamental and derived quantities we shall find a very natural method of correlating the physical quantities having intensive magnitude with those having extensive magnitude, since we shall find it possible to define all our physical quantities in terms of a set all of which have extensive magnitude. This procedure, which has not hitherto been adopted, seems to be a distinct advance in the direction of simplicity and fundamentality.

§ 4. On Fundamental and Derived Quantities.

It is not necessary to take all of the many different kinds of quantity used by the physicist as indefinable, since most of them can be defined in terms of a few which may be taken as fundamental. Thus it is customary to regard a quantity of velocity as a quantity of length divided by a quantity of time, a quantity of volume as the product of three quantities of length, a quantity of density as a quantity of mass divided by a quantity of volume, a quantity of momentum as a quantity of mass multiplied by a quantity of velocity, etc. In definitions of this kind the operations "multiplied by," "divided by," etc., may be taken as further indefinables and we shall present a brief discussion of them in a following section.

[1] There is one case in which the method for measuring quantities with extensive magnitude described above seems to fail. Time intervals are quantities which have extensive magnitude, since we can obviously regard a twenty-minute interval as composed of two ten minute intervals following one another. It is obvious, however, that, for us, a direct experimental comparison of one time interval with another is impossible, and hence we cannot measure a time interval by fitting together unit time intervals until we obtain the magnitude in question. For this reason some artificial method of measuring time, such as counting the beats of a pendulum, has to be devised. A direct comparison of time intervals might be possible to an observer with a chrono-synoptic point of view.

The relation between a derived quantity and the fundamental kinds of quantity in terms of which it is defined is customarily symbolized by physicists by a statement of the so-called dimensions of the quantity in question. Thus velocity is said to have the dimensions of length divided by time and this is symbolized by the formula $[lt^{-1}]$, and momentum which is defined as mass times velocity is said to have the dimensional formula $[mlt^{-1}]$. *The dimensional formula of a quantity may be regarded as a shorthand statement of the definition of that kind of quantity in terms of the kinds of quantity chosen as fundamental, and hence also as a partial statement of the "physical nature" of the quantity in question.*

§ 5. CRITERIA FOR THE CHOICE OF FUNDAMENTAL QUANTITIES.

In choosing a set of fundamental kinds of quantity, it will be desirable to keep in mind the following considerations, namely, (1) that the number of kinds chosen shall be *sufficient* (and not redundant) for the part of physical science that is to be handled; (2) that all of the kinds chosen as fundamental shall have *extensive* magnitude; and (3) that the choice shall be such as to further the requirements of simplicity.

Sufficiency.—For the body of physics which is at present well established we seem to need five kinds of fundamental quantity for defining all the quantities in use. The ones usually chosen for this purpose are those of length l, time t, mass m, magnetic permeability μ, and temperature T, although, as we shall later see, a somewhat different set of five will be more satisfactory. These five indefinables, together with the indefinable relations "multiplied by" and "divided by," etc., of which we have already spoken, are sufficient and also apparently necessary for defining all the quantities we need at present.[1]

The reason that just five different kinds of fundamental quantity are necessary seems to be because the science of physics is at the present time considering five fundamentally different kinds of "thing," namely, space, time, matter, electricity, and entropy (or "degree of run-downness"). The nature of these different kinds of thing will perhaps be clearer if we point out that we can arrange the physical sciences in a hierarchy such that each successive member introduces the consideration of one additional kind of "thing." Thus the science of *geometry* considers one kind of "thing" space, *kinematics*, which is the science of moving points, lines, planes, etc., introduces the further "thing" time, *mechanics*, which considers moving bodies rather than immaterial points and lines, introduces the new "thing" matter, *electrodynamics*, which

[1] It was first pointed out by Sir A. W. Rücker, Proc. Phys. Soc., London *10*, 37 (1888) that *five* kinds of quantity are necessary, it having previously been assumed that length, time and mass would be sufficient for the whole of physics.

discusses the further laws of motion when the bodies may carry charges of electricity, introduces electricity as a further "thing," while *thermodynamics*, which considers systems composed of such a large number of individual units that we cannot follow the behavior of the separate parts but only the statistical results of their interactions, introduces a new "thing" entropy or "degree of run-downness."

Extensivity of Fundamental Quantities.—Our second criterion for the choice of fundamental quantities was that they should all have extensive magnitude. This is desirable since we shall then be able, as we have already seen, to measure all our fundamental kinds of quantity by a simple process of fitting unit quantities together until we reach the magnitude of the quantity which we are measuring. Moreover, since all further kinds of quantity will be directly defined in terms of this fundamental set, we shall then be in a position to measure all the derived quantities by determining the magnitudes of the fundamental quantities on which they depend. This will be particularly important in the case of derived quantities having intensive magnitude, since otherwise we should have to devise some purely arbitrary method of correlating the intensive magnitude, involved with quantities having extensive magnitude. These considerations alone are enough to make us reject permeability (or dielectric inductivity) and temperature as suitable fundamental quantities.

Simplicity.—Our third criterion for choosing fundamental quantities was simplicity, which includes the following considerations: (1) that the kinds of quantity should be psychologically simple to grasp; (2) that they should permit straightforward definitions of the desired derived quantities; and (3) that they should bear as simple a relation as possible to the five different kinds of "thing" of which we have spoken in a previous paragraph.

Let us consider an actual set of fundamental kinds of quantity which fulfills the requirements.

§ 6. FUNDAMENTAL QUANTITIES.

For *spatial considerations* we shall choose *length* as the fundamental kind of quantity. This is the choice ordinarily made by physicists. It has extensive magnitude, is an idea relatively simple to grasp, permits a straightforward definition of such quantities as area and volume, and bears a simple relation to the corresponding fundamental "thing"—space. We see that volume would be a less successful choice, since its three-fold nature makes it a psychologically less simple concept, and furthermore the definition of length in terms of volume would be much less straightforward than the reverse procedure.

For *temporal considerations* we shall choose *time interval* as the fundamental kind of quantity, a choice which is obvious in the light of the criteria which we have laid down.[1]

For *mechanical considerations* we shall make the usual choice of *mass* as the fundamental kind of quantity. This choice conforms to all the criteria that we have laid down; in particular we should not overlook the intimate relationship between the "thing" matter and mass, the latter often being considered indeed as a measure of the quantity of matter. The close connection between mass and energy found in the newer mechanics is also significant.

For *electrical considerations* we shall choose *quantity of electric charge* as fundamental. This is superior to the usual choice of magnetic permeability (or dielectric inductivity) not only because it has extensive magnitude but also because of greater psychological simplicity and a more direct relation to the fundamental "thing" electricity. It is interesting to note that the recent experimental work of Millikan and others has indicated that electric charge is in the group of quantities already spoken of which have *discrete* magnitudes. The fundamental unit of electric charge is that of one electron and the most natural method of measuring a charge would be to count the number of electrons which it contains. Such a procedure presents of course experimental difficulties, and since we usually deal with charges containing a large number of electrons we can if desired treat these quantities as though they had a continuous series of magnitudes using any convenient unit. This is analogous to measuring apples by the peck instead of by counting.

For *thermodynamic* considerations we shall choose *entropy* as the fundamental kind of quantity, although the choice has previously been temperature. Perhaps this choice of entropy can not be justified on the ground of psychological simplicity since temperature certainly seems to most persons a simpler idea. Entropy, however, does bear a simpler relation to the kind of "thing" considered in thermodynamic considerations since it may be regarded as a quantitative measure of the "degree of run-downness" of a system. Entropy has, moreover, extensive magnitude since the entropy of a system is the sum of the entropies of its parts, while the temperature of a system is certainly not the sum of the temperatures of its parts.

§ 7. On the Indefinable Operations.

Before proceeding to the actual definition of the derived quantities of physics, in terms of the fundamental quantities, we must first turn

[1] We have already pointed out the experimental difficulty of fitting unit quantities of time together.

our attention to the indefinable *operations* which enter into the definitions.

We have already seen that quantities are to be regarded as magnitudes which have been particularized either spatially or temporally or both. It should be noticed now that in the case of length, which is one of our five fundamental kinds of quantity, a spatial particularization of a magnitude of length gives us a *vector* quantity, since a complete particularization will state not merely the location in space but the direction and sense of the quantity of length in question. For this reason, in defining derived quantities in terms of fundamental quantities we find that we need to include among our operations the two types of vector multiplication.

Our indefinable operations will then be multiplication, division and differentiation and the two types of vector multiplication which lead respectively to the inner and outer product. Multiplication may be indicated by a simple juxtaposition of the quantities involved, thus ab; division by the superposition of dividend and divisor, thus a/b; differentiation in the familiar way, thus da/db; the formation of the inner product by a small dot between the quantities involved, thus $\boldsymbol{a}\cdot\boldsymbol{b}$; and the formation of the outer product by the interposition of a small cross, $\boldsymbol{a}\times\boldsymbol{b}$.[1]

We shall take these operations for our purposes as indefinable, of course, however, without in the least objecting to their being defined in terms of something simpler in some more fundamental discussion. Certain characteristics of the five operations may be noted. In the case of multiplication one of the multipliers must be a scalar quantity, in the case of division the divisor must be a scalar, and we shall only carry out differentiation with respect to scalar quantities. In forming the inner or outer product both quantities must be vectors (or pseudo scalars). In accordance with Lewis[2] the outer product of two lines will be the parallelogram which they determine rather than a line perpendicular thereto, and the outer product of three lines will be the corresponding parallelopiped. It should also be particularly noticed that these five operations are to be considered as performed upon actual quantities, not upon numbers which are used to represent the magnitudes involved.

§ 8. The Definition of Derived Quantities.

We are now in a position to proceed to the definition of the quantities of physics in terms of the five fundamental quantities and the five

[1] Vector quantities are indicated by heavy Clarendon type.

[2] Lewis, Proc. Amer. Acad., *46*, 165 (1910).

indefinable operations. It will be sufficient to give one or two examples of such definitions for each of the general classes of geometrical, kinematical, mechanical, electrical and thermodynamic quantities.

As an example of *geometrical* quantities consider area. Two lines running from a given point determine a parallelogram, and we define the quantity of area of the parallelogram as the outer product of the two quantities of length, thus $\boldsymbol{A} = \boldsymbol{l}_1 \times \boldsymbol{l}_2$. In accordance with the rules of vector analysis, the numerical measure of the area $\boldsymbol{A}$ will be the product of the numerical measures of the two lines multiplied by the sine of the included angle.

This example illustrates an important characteristic of derived quantities, namely, that they are themselves particularized spatially or temporally. This is of course perfectly proper since we have from the start looked upon quantities as particularized magnitudes. And hence it need not surprise us that two magnitudes of length which have a definite location in space and in time should determine a magnitude of area which is also definitely located in space and in time.

As an example of *kinematical* quantities we may take velocity. The quantity of velocity of a moving point will be defined as the limit approached by the quantity of length travelled by the quantity of time required as we make the time interval smaller and smaller, thus

$$\boldsymbol{v} = \lim_{t \to 0} \left(\frac{\boldsymbol{l}}{t} \right) = \frac{d\boldsymbol{l}}{dt}.$$

Here again we see that derived quantities are themselves particularized spatially or temporally.

As examples of *mechanical* quantities we may define the quantity of momentum of a particle as its quantity of mass multiplied by its quantity of velocity, $\boldsymbol{M} = m\boldsymbol{v}$, and the quantity of force acting on the particle as the differentiation with respect to the time of the quantity of momentum $\boldsymbol{F} = (d/dt)(m\boldsymbol{v})$. In the definition of work we introduce for the first time the use of the inner product. The quantity of work done by a force acting at a given point being the inner product of the quantity of force by the quantity of length through which the point is displaced, thus $W = \boldsymbol{F} \cdot \boldsymbol{l}$, or in the case of variable forces $dW = \boldsymbol{F} \cdot d\boldsymbol{l}$. In accordance with the rules of vector analysis, the numerical measure of the work is equal to the product of the numerical measures of the force and the displacement multiplied by the cosine of the included angle.

As a single example of *electrical* quantities, we define the electric potential at a point as the work done in bringing up a small charge to the point in question from an infinite distance divided by the charge itself.

The test charge is supposed taken small enough so that it will not itself disturb the electrical field.

As the most important example of thermodynamic quantities, we define the quantity of temperature of a given system by the equation $T = dQ/dS$ where dS is the infinitesimal change in the quantity of entropy of the system corresponding to the absorption of heat energy dQ, quantities of energy having already been defined in terms of mechanical quantities.

Without presenting any further complete definitions we give here for future reference a table of the "dimensions" of physical quantities

TABLE I.

Dimensions of a Few Important Quantities.

	Dimensions.		Dimensions.
Geometrical Quantities:		*Electrical Quantities:*	
Length, l	$[l]$	Charge, e	$[e]$
Area, A	$[l^2]$	Potential	$[ml^2t^{-2}e^{-1}]$
Volume, V	$[l^3]$	Capacity	$[m^{-1}l^{-2}t^2e^2]$
Angle	$[l^0]$	Inductivity, K	$[m^{-1}l^{-3}t^2e^2]$
Kinematical Quantities:		Current, I	$[lt^{-1}e]$
Time, t	$[t]$	Resistance, R	$[mlt^{-1}e^{-2}]$
Velocity, v	$[lt^{-1}]$	Electric field strength, E	$[mlt^{-2}e^{-1}]$
Acceleration, a	$[lt^{-2}]$	Magnetic field strength, H	$[l^{-1}t^{-1}e]$
		Magnetic pole strength, M	$[ml^2t^{-1}e^{-1}]$
Mechanical Quantities:		Permeability, μ	$[mle^{-2}]$
Mass, m	$[m]$	Magnetic induction, B	$[mt^{-1}e^{-1}]$
Density	$[ml^{-3}]$		
Momentum, M	$[mlt^{-1}]$	*Thermodynamic Quantities:*	
Force, F	$[mlt^{-2}]$		
Energy, U	$[ml^2t^{-2}]$	Entropy, S	$[S]$
Energy Density, u	$[ml^{-1}t^{-2}]$	Temperature, T	$[ml^2t^{-2}S^{-1}]$
Pressure, p	$[ml^{-1}t^{-2}]$	Specific heat	$[m^{-1}S]$
Elasticity of compression, $E = V(dp/dV)$	$[ml^{-1}t^{-2}]$		

which are shorthand restatements of the way the derived quantities depend on the fundamental ones and may be looked upon as statements of the physical nature of the quantities in question. The expressions given in the table are not completely satisfactory since they do not distinguish between the different kinds of product nor take account of the frequent necessity of using differential quantities.[1] They are, however, sufficient for many purposes.

[1] It was first pointed out by W. Williams, Proc. Phys. Soc. Lond., *11*, 257 (1892) that ordinary dimensional formulæ do not state all that we should be glad to have them since they omit any reference to the vector nature of some of the quantities. He proposed to remedy this by using different symbols for distances in different directions.

§ 9. UNITS OF MEASUREMENT.

Units for Fundamental Quantities.—In picking out units for the measurement of the five fundamental kinds of quantity considerations as to convenience in magnitude, ease of comparison, permanence and reproducibility of the standards and generality of use will ordinarily be the determining factors. And in agreement with these considerations, the system which is universally adopted by scientists takes as its five fundamental units the centimeter, the second, the gram, the so-called magnetic unit of electric quantity and the centigrade degree. As pointed out by Planck,[1] however, this particular choice has been unduly influenced by the accidents of our purely terrestrial civilization. It would seem to the writer a rational choice to pick out the three units of length, time and mass to satisfy the three conditions that light should travel unit distance in unit time, that two electrons unit distance apart should repel each other with unit force, and that there should be unit energy in an energy quantum having unit frequency, to take as unit charge of electricity that of the electron, and to make unit increase in entropy that which occurs per molecule when a perfect gas is expanded without doing external work to e times its original volume.[2]

Units for Derived Quantities.—The units for measuring derived quantities may be chosen if desired without any reference to the fundamental units, on the basis of considerations of convenience or familiarity. In case the derived quantity has intensive rather than extensive magnitude some more or less artificial correlation of the magnitude in question with quantities having extensive magnitude will then have to be used, as has been done in the case of our ordinary temperature scale.

A more rational device, however, for the measurement of derived quantities will be to give the unit the magnitude which is obtained by combining unit quantities of the fundamental kinds in the way indicated by the dimensional formula. Such a choice simplifies the theory of measurement of derived quantities since it is then possible to measure

[1] "Theories der Wärmestrahlung," Leipzig, 1913.

[2] This choice of units would give the numerical value of unity to the velocity of light c, to the constant K in Coulomb's law for the repulsion of electric charges, to the constant h of the quantum theory and to the "gas constant for one molecule," $k = R/N$. Making these constants unity, will result in considerable simplification in numerical calculations.

This choice of units differs from that of Planck in that he also makes the gravitation constant equal to unity. With such a choice, however, it is not possible to take the charge of the electron as unity and at the same time have the constant of Coulomb's law equal to unity, and it seems preferable to sacrifice the gravitational constant rather than that of Coulomb's law, since the author's development of the theory of similitude has indicated that the gravitational constant may not be very fundamental. See Tolman, PHYS. REV., 3, 246 (1914); Nordström, Ofversigt af Finska Vetenskaps-Societens Förhadnlinger, Afd. A, 57, 1 (1914–15).

the fundamental quantities upon which the derived quantity depends and combine the numbers obtained in the way indicated by the dimensional formula. In the case of quantities having intensive magnitude this is particularly significant since all the fundamental kinds of quantity have been picked out so as to have extensive magnitude, and we obtain in this way a natural rather than an arbitrary correlation between the intensive magnitudes to be measured and a set of extensive magnitudes.

On the Confusion of Quantity and Unit.—The fact that it has become usual to pick out the units for derived quantities in the way indicated has sometimes led to an unfortunate confusion as to the real significance of dimensional formulæ. Thus there has grown up the practice of speaking of the dimensions of a unit when what is really intended is the dimensions of the quantity involved. It certainly seems best, however, to use the dimensional formula of a quantity as a shorthand restatement of its definition in terms of the fundamental kinds of quantity. The dimensional formula is thus a symbol for the physical nature of the derived quantity and a recapitulation of the *necessary* relation between different kinds of quantity rather than the statement of a relation between units which we find convenient.

This opportunity for confusion has been particularly troublesome in the case of the fundamental kinds of quantity, since the desirability of making the units for the different fundamental kinds of quantity depend upon each other in the way indicated has sometimes led to an unfortunate desire to try to make the dimensions of the *quantities* involved depend upon each other in the same way that we have found convenient for the *units*.

The most common proposal of this kind has been to try to give time the same dimensions as length. This has arisen partly, as we have just pointed out, because of the real desirability of making the unit of time depend on the unit of length, and also partly because the recent developments connected with the theory of the relativity of motion have shown the frequent convenience of plotting kinematical events in a four-dimensional space in which the time axis is taken at right angles to the three axes of ordinary space, which has apparently led to the conclusion that time and space must then have the same physical nature and hence should have the same dimensions.

It seems to the writer, however, erroneous, to give time and length the same dimensions. We have already pointed out that a relation between units of measurement which may be found convenient should not be confused with that necessary relation between kinds of quantities which is symbolized by dimensional formulæ. And as to the possibility

of plotting time and distance along axes at right angles to each other this is no more significant than the possibility of plotting the pressures and volumes of a perfect gas in a two-dimensional space, a procedure which certainly does not make pressure have the dimensions of volume. Indeed the fact that, for the most successful plotting of kinematical events, it is necessary to use a non-Euclidean space in which the properties along the time axis are quite different from those along a space axis should have further emphasized the difference between the physical nature of space and time and hence the necessity for difference in dimensions.

In particular we may also point out that the assignment of the same dimensions to time and length would greatly reduce the value and specificity of the principle of the dimensional homogeneity of physical equations.[1] Consider as a single example, the use of the principle of dimensional homogeneity to determine the way in which the centripetal force f which constrains a particle to move in a circle depends on the mass of the particle m, its velocity v and the radius of the circle r. We have

$$f = \phi\,(m, v, r),$$

where ϕ is the function whose form we wish to determine. Now it is evident that the desired combination of m, v, and r must have the physical nature of a force and hence the dimensions of force. The three quantities involved have the dimensions $[m] = [m]$, $[v] = [lt^{-1}]$, $[r] = [l]$ and it is evident that the only combination of these which has the dimensions of force, $[mlt^{-2}]$, is $[mv^2/r]$ and hence the desired equation must be of the form

$$f = k\frac{mv^2}{r},$$

where k is some purely numerical constant which turns out to be unity with the usual choice of units.

Suppose now, however, we had given time and length the same dimensions l. Then we should have had $[f] = [ml^{-2}]$, $[v] = [l^0]$, $[r] = [l]$, and we see that any one of an infinite number of combinations m/r, mv/r, mv^2/r, mv^n/r, etc., would satisfy the requirements of dimensional homogeneity, and such an identification of the dimensions of length and time would be disastrous to the import of the principle of dimensional homogeneity.

I am inclined to believe that this same confusion of the dimensions of a *quantity* with what might be called the dimensions of the *unit* which

[1] For a discussion of the principle of dimensional homogeneity, see for example the writer's article, PHYS. REV., *6*, 219 (1915).

we have chosen for its measurement is a contributory cause for a number of criticisms which have been made on the principle of similitude.[1]

Thus I am quite willing to agree with Professor Bridgman[2] when he says that "the number of fundamental *units* we are to use is in large part a matter of convenience. We can get along with fewer units if we introduce corresponding definitions which are in accord with experimental facts." I could not agree with him, however, if he should conclude from this that the number of fundamental kinds of *quantity* is also reduced, merely by setting up convenient relations between their units. And this would seem to be the basis for what seems to me his erroneous belief that the reasoning used by the theory of similitude is nothing but ordinary dimensional reasoning based on a set of dimensions with only one fundamental kind of quantity. If this contention of Professor Bridgman's were correct, then as will be seen by an examination of the transformation equations involved, we should not only have to ascribe to time the dimensions of length, but to mass the dimensions of the reciprocal of a length, to electric charge the dimensions of angle, and to temperature the dimensions of the reciprocal of a length. These startling conclusions would certainly disagree with our idea of the dimensions of a quantity as a shorthand restatement of its definition and hence as an expression of its essential physical nature, and if such a set of dimensions were adopted, the principle of dimensional homogeneity would become meaningless, since this principle merely expresses the obvious necessity that all the terms in an equation connecting physical quantities shall have the same physical nature.

By way of digression, it may be pointed out from quite a different angle, that the principle of similitude most certainly contains elements foreign to the theory of dimensions, since as was shown in my first paper it permits the prediction that Newton's law of gravitation is not a complete statement of the experimentally determinable facts. If the principle of similitude is true, we are led either to the conclusion that "gravitational mass" is not the same thing as ordinary mass, or to the conclusion that there is a gravitational mechanism of such a nature that the gravitational constant is not independent of other physical quantities. We have perhaps already received the first hint of experimental evidence confirming this prediction if the elaborate work of Shaw[3] showing the effect of temperature on gravitation should prove to be conclusive. Gravitational work is also being planned in this laboratory. A theory

[1] Tolman, PHYS. REV., *3*, 246 (1914); Ibid., *4*, 145 (1914); Ibid., *6*, 219 (1915); Ibid., *8*, *8* (1916).

[2] Bridgman, PHYS. REV., *8*, 423 (1916).

[3] Shaw, Roy. Soc., Phil. Trans., *216*, 349 (1916).

should certainly be evaluated not only on the basis of its success in correlating known facts but also for its success in pointing the way to further research.

§ 10. FUTURE DEVELOPMENTS.

In conclusion it will not be out of place to consider certain possibilities of future development in the theory of quantities.

Increase in the Number of Fundamental Kinds of Quantity.—As science becomes more thoroughly rationalized in its more complex fields we shall undoubtedly increase the number of fundamental kinds of quantity. This may take place and indeed has done so in the past in two different ways.

In the first place, as physics develops we may discover new fundamental kinds of "thing" to which we shall ascribe objective existence in the external world and shall then need quantities for measuring this new kind of "thing." Thus in the past only three kinds of quantity, those of mass, length and time, were taken as fundamental, and it was only with the later development of electrical science that we conceived of another kind of "thing" besides matter, namely, electricity, as existing in the world of space and time. To the writer, as pointed out above, it does not seem at all impossible, for example, that we may in the future find "gravitational mass" to be a different kind of thing from ordinary mass, and only accidentally proportional to it in the materials with which we have hitherto experimented. If this should prove to be the case we shall then need a new fundamental kind of quantity for measuring the new kind of thing "gravitational mass."

A second possibility of increasing the number of fundamental kinds of quantity can arise from the desirability of looking at phenomena from a new point of view, with the consequent *invention* of a new fundamental kind of thing. Thus thermal phenomena can apparently all be explained by a consideration of the behavior of the individual atoms and molecules or other elements of which material systems are composed, and as long as we look at these systems from such a microscopic point of view (for example that of the kinetic theory) we do not need to introduce other fundamental kinds of quantity than those of mass, length, time and electricity. But when we wish to look at these systems from a macroscopic point of view, that is, to neglect the behavior of individual molecules and obtain laws which express the statistical result of their interactions, we objectify a new fundamental kind of quantity, entropy or "degree of run-downness." A bridge between the two points of view is given by the Boltzman-Planck equation connecting entropy S

and probability of configuration W,

$$S = k \log W.$$

It should also be noted that as other fields of science besides physics in the narrower sense come into the field of mathematical analysis we have to introduce further kinds of quantity some of which will probably be most conveniently taken as fundamental and other as derived. Thus in chemistry we need, for example, different kinds of quantity for measuring amounts of each chemical species, and in biology we begin to find the introduction of new kinds of quantity when, for example, the vitality of a given strain of bacteria is correlated with the time it takes for a definite inoculation to produce a definite amount of chemical change. And in the distant future we may even hope that the speculations of economists, and sociologists will at least regard exact quantitative investigation as a desired goal and such developments will also necessitate the introduction of new kinds of measurable quantity.

Possibility of Decreasing the Number of Fundamental Kinds of Quantity.—In building up any logical system it is of course desirable to choose only a small number of propositions as postulated and only a small number of concepts as indefinable. For this reason it would be desirable to decrease the number of kinds of quantity that we take as fundamental and indefinable, provided this did not itself introduce complexities. At the present time, however, the writer can see no possibility of reducing the number of fundamental kinds of quantity for the accepted body of physics below five, without, indeed, producing a complete alteration in the nature of our system of physical concepts. The reason why there must be five fundamental kinds of quantity is because the physicist has chosen to make use of five fundamental kinds of concept, namely space, time, matter, electricity, and entropy or probability of configuration. In terms of these five kinds of "thing" he can define other "things" in he physical world. T hus he can define a quantity of velocity as a quantity of length divided by a quantity of time, or a quantity of momentum as a quantity of mass multiplied by a quantity of velocity, but if he tries to claim that a quantity of time is the same thing as a quantity of length, or that a quantity of mass is the same thing as the reciprocal of a quantity of length he will meet difficulties that are insurmountable.

UNIVERSITY OF ILLINOIS,
December 9, 1916.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

AN EXTENSION OF THE MAYER EXPERIMENT.[1]

BY R. R. RAMSEY.

THE experiment consists in giving the bicycle balls which are floated on mercury a rotation and thus better representing the modern idea of atomic structure than is done by the floating needles. The apparatus consists of a wooden tray with a platinum electrode in the middle and four platinum electrodes, one in each corner. When a current of electricity is sent into the mercury through the middle electrode and out at the corner electrodes which are connected in multiple, we have an approximately radial current flowing at right angles to the magnetic field of the magnet which plays the part of the positive nucleus. This causes the mercury to rotate and the balls are carried with the mercury. The motion of the balls is not simply the rotation of the Mayer figures. The centrifugal force and the fact that the angular velocity decreases as the radius increases causes the motion of any one ball to be very complicated. With two balls the paths of the balls approximate to rotating ellipses, the balls taking turns at the center. If the balls represent electrons the various motions and perturbations may be said to be the source of the various radiations of the atoms. A ball coming from the outside into the system produces a great disturbance. This disturbance may be said to be the source of the X-rays. At times a ball starts out on a tangent and seems to try to escape. A tray was made in which a ring of iron was imbedded, thus causing the magnetic field of the magnet to vary from a maximum at the center and pass through a minimum and then increase to a maximum over the ring, as we move along a radius. By this means a ball which starts on a tangent is held over the ring. If the balls represent alpha particles then we have an illustration of the disintegration of a radio-active substance. From five to ten disintegrations can be represented in this manner.

INDIANA UNIVERSITY,
Nov. 11, 1916.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

THE MAGNETIC PROPERTIES OF FE, NI, AND CO ABOVE THE CURIE POINT, AND KEESOM'S THEORY OF MAGNETIZATION.[1]

BY EARLE M. TERRY.

THE paper gives the final results of an investigation of the magnetic properties of iron, nickel and cobalt from the temperature at which ferro-magnetism disappears up through the melting point, a preliminary report of which was made at the Chicago meeting of last year. The work has been carried out in a good vacuum, the high temperatures being obtained by means of a special tungsten furnace. The results show that Curie's law,

$$x \cdot T = \text{const},$$

where x is the susceptibility and T the absolute temperature, holds in the case of nickel only, and here in but a restricted region. For β iron, β cobalt and β nickel in the lower temperature region, the $1/x$, T curves all show a distinct concavity upward instead of being straight lines, as demanded by Curie's law. For γ iron, the susceptibility varies only slightly with temperature but the concavity of the x, T curve is downward instead of upward as required by the law. At the γ–δ transformation point an increase in x of approximately 50 per cent. occurs above which the x, T curve is a straight line with a large negative slope.

In passing through the melting point, the susceptibility for iron decreases rapidly, while for nickel there is no change whatever, but for cobalt the x, T curve shows a slight depression.

These results are in marked contrast to those of Weiss and Foëx who found Curie's law to hold in all cases. Keesom's quantum theory of magnetization, including the assumption of a zero-point energy, gives x, T curves of the same general character as those obtained by the author but they depart from the asymptote less rapidly than required by experiment. An attempt has been made to obtain a better fit by modifying the theory by the assumption of a larger zero point energy, but it has been found thate ven when the latter is taken twenty times that of the theory, the agreement with experiment is still unsatisfactory.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN.

A. C. AND D. C. CORONA IN HYDROGEN.[1]

BY JOHN W. DAVIS.

THE corona discharge between concentric cylinders when the space between the cylinders is filled with hydrogen presents some characteristic differences from the discharge in air.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

1. The discharge in hydrogen does not start and stop at the same voltage.

2. The current obtained with a given voltage impressed between the two cylinders depends on whether that value of voltage was obtained by an approach through lower or higher values.

3. The discharge takes place at decidedly lower voltages in hydrogen than in air.

4. There is a wide difference in the critical electric intensity at the surface of the inner cylinder, depending on whether this cylinder is positive or negative.

5. The character of the volt-ampere characteristics with the inner cylinder positive and the inner cylinder negative are totally different.

6. The corona discharge in hydrogen may be used for the rectification of alternating currents.

7. The appearance of the corona, spark and arc are different in air and hydrogen.

8. There is apparent evidence of ionization potential gradients at the surface of both the inner and outer cylinders, as is shown by the appearance of light at both of these surfaces.

PHYSICAL LABORATORY,
UNIVERSITY OF ILLINOIS.

THE KINETIC THEORY OF NON-SPHERICAL RIGID MOLECULES.[1]

BY YOSHIO ISHIDA.

THIS paper deals with the kinetic theory of a system of non-spherical rigid molecules according to the method of Boltzmann's integro-differential equation.

With use of the general notion of invariants of encounters, the conditions of conservation during encounter, which are independent of any supposition regarding the frequency of various types of encounter, are shown to lead to a kind of extended hydrodynamical theory, which includes not only the familiar equations of mass, force and energy but in addition what may be called the equation of gyroscopic disturbances.

Introduction of a plausible hypothesis regarding the frequency of encounters leads to the conclusion that for the case of equilibrium a system of centrally symmetric rigid molecules would satisfy the same relation of pressure, volume and temperature as the ideal monatomic gas.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
November 17, 1916.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.